



University of Pennsylvania
ScholarlyCommons

Publicly Accessible Penn Dissertations

2021

Essays On Financial Crises

Sergio Villalvazo Martin
University of Pennsylvania

Follow this and additional works at: <https://repository.upenn.edu/edissertations>



Part of the [Economics Commons](#)

Recommended Citation

Villalvazo Martin, Sergio, "Essays On Financial Crises" (2021). *Publicly Accessible Penn Dissertations*. 4093.

<https://repository.upenn.edu/edissertations/4093>

This paper is posted at ScholarlyCommons. <https://repository.upenn.edu/edissertations/4093>
For more information, please contact repository@pobox.upenn.edu.

Essays On Financial Crises

Abstract

This dissertation studies financial crises of the Sudden Stop type where large reversals in the current account are triggered by a deflation mechanism that tightens the borrowing capacity of individuals, and amplifies the effects of negative shocks. These episodes are characterized by large drops in consumption and domestic asset prices. The first chapter argues that inequality in wealth and leverage across households plays an important role in determining the aggregate effects of a crisis. Next, the second chapter studies the role that foreign direct investment flows have on the different frequency of crises observed in advanced and emerging economies. Finally, the third chapter develops a new algorithm that allows solving dynamic stochastic general equilibrium (DSGE) models with occasionally-binding constraints much faster than existing methods.

Degree Type

Dissertation

Degree Name

Doctor of Philosophy (PhD)

Graduate Group

Economics

First Advisor

Enrique G. Mendoza

Second Advisor

Frank Schorfheide

Keywords

Asset-pricing, Debt-deflation, Financial crises, Household leverage, Inequality, Sudden Stops

Subject Categories

Economics

ESSAYS ON FINANCIAL CRISES

Sergio Villalvazo Martín

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2021

Co-Supervisor of Dissertation



Enrique G. Mendoza
Presidential Professor of Economics

Co-Supervisor of Dissertation



Frank Schorfheide
Professor and Chair of Economics

Graduate Group Chairperson



Jesús Fernández-Villaverde
Professor of Economics

Dissertation Committee

Dirk Krueger

Walter H. and Leonore C. Annenberg Professor in the
Social Sciences and Professor of Economics and Finance

Alessandro Dovis

Assistant Professor of Economics

ESSAYS ON FINANCIAL CRISES

Copyright ©

Sergio Villalvazo Martín

2021

To my parents

ACKNOWLEDGMENTS

I am immensely grateful to my advisors, Enrique G. Mendoza and Frank Schorfheide, and my dissertation committee, Alessandro Dovis and Dirk Krueger, for their exceptional supervision and constant support. I also want to thank Hal Cole, Frank Diebold, Jesús Fernández-Villaverde, Jeremy Greenwood, George Mailath, Steven Matthews, Guillermo Ordoñez, Andrew Postlewaite, and José-Victor Ríos-Rull for all your time inside and outside the classroom. Having the opportunity to learn from such an amazing pool of teachers and mentors has made me a better economist.

I want to thank Geo for being by my side and making me a better person. I thank my family for always believing in me and supporting me. My parents, Elsa and Sergio, and sister, Mariana; you are an example I look up to. You are my source of inspiration and I thank you with all my heart.

I want to thank all the friends that I met at the University of Pennsylvania for making the Ph.D. journey an incredible time. Gabrielle, Gorkem, Joao, and Tomas are the best classmates anyone could ask for. Thanks to André, Camille, David, Eugenio, Ewelina, Hanbaek, María José, Minsu, Montse, Omer, Paul, and Roberto for sharing this entire ride.

Lastly, I am grateful to the Money-Macro Club participants at Penn for their continuous feedback, and to the Federal Reserve Bank of Atlanta for their hospitality while writing the first chapter of this dissertation. All remaining errors are my own.

ABSTRACT

ESSAYS ON FINANCIAL CRISES

Sergio Villalvazo Martín

Enrique G. Mendoza and Frank Schorfheide

This dissertation studies financial crises of the Sudden Stop type where large reversals in the current account are triggered by a deflation mechanism that tightens the borrowing capacity of individuals, and amplifies the effects of negative shocks. These episodes are characterized by large drops in consumption and domestic asset prices. The first chapter argues that inequality in wealth and leverage across households plays an important role in determining the aggregate effects of a crisis. Next, the second chapter studies the role that foreign direct investment flows have on the different frequency of crises observed in advanced and emerging economies. Finally, the third chapter develops a new algorithm that allows solving dynamic stochastic general equilibrium (DSGE) models with occasionally-binding constraints much faster than existing methods.

TABLE OF CONTENTS

DEDICATION	iii
ACKNOWLEDGMENTS	iv
ABSTRACT	v
TABLE OF CONTENTS	viii
LIST OF TABLES	x
LIST OF ILLUSTRATIONS	xii
1 Inequality and Asset Prices during Sudden Stops	1
1.1 Introduction	1
1.2 Related Literature	5
1.3 The Cross-Sectional Effects in the Data	8
1.3.1 Description of the data	9
1.3.2 Stylized Facts: Differentiated Individual Effects	11
1.3.3 Stylized Facts: Heterogeneous Consumption Dynamics	13
1.4 Model	16
1.4.1 Environment	16
1.4.2 Households	18

1.4.3	Exogenous stochastic processes	19
1.4.4	Closing the domestic asset market	20
1.4.5	Recursive Formulation	20
1.5	The Cross-Sectional Effects in the Model	23
1.5.1	Market Incompleteness and Risk Exposure	23
1.5.2	Financial Premia	25
1.6	Quantitative Analysis	29
1.6.1	Calibration	30
1.6.2	Stationary Model	35
1.6.3	Aggregate Risk Model	44
1.7	Conclusions	56
	References	58
1.A	Appendix	62
1.A.1	The 2009 Mexican Sudden Stop at the Aggregate Level	62
1.A.2	Solution Algorithm	63
2	FDI Flows and Sudden Stops in Small Open Economies	66
2.1	Motivation	66
2.2	Empirical Evidence	72
2.2.1	Differences in the FDI flows and the capital stock	73
2.2.2	Importance of the international volatility	79
2.3	Model	80
2.3.1	Environment	80
2.3.2	Recursive competitive equilibrium	84
2.4	Quantitative Analysis	87
2.4.1	Calibration	87

2.4.2	Quantitative results	89
2.4.3	Impulse response analysis	96
2.4.4	Anecdotal evidence: episodes of expropriation	97
2.5	Conclusions	99
	References	102
2.A	Appendix	105
2.A.1	Description of the data	105
3	<i>FiPIt</i>: A Simple, Fast Global Method for Solving Models with Two	
	Endogenous States & Occasionally Binding Constraints	108
3.1	Introduction	108
3.2	A Fixed-Point Iteration Algorithm for a Simple Savings Model	112
3.3	The <i>FiPIt</i> Method for Two-Dimensional Models	118
3.3.1	Model structure and equilibrium conditions	118
3.3.2	Description of the <i>FiPIt</i> algorithm	123
3.4	Application to Sudden Stops Model	133
3.4.1	Comparison of Results & Performance Metrics	134
3.4.2	Robustness Analysis & Credit Constraint Variations	145
3.5	Conclusions	148
	References	153
3.A	Appendix	155
3.A.1	Introduction	155
3.A.2	Recursive Equilibrium Conditions	157
3.A.3	Contents of the mainFiPIt.m program	158
3.A.4	Auxiliary Notes	168
3.A.5	Sketch of Other <i>FiPIt</i> Applications	170

LIST OF TABLES

Inequality and Asset Prices during Sudden Stops	1
1.1 Mean net wealth and its composition by deciles in 2005	10
1.2 Median % Real Estate Change 2005-09	12
1.3 Median % Consumption Change	13
1.4 Distribution of Households in %	15
1.5 Parameters	30
1.6 Annual Income Process Estimates	33
1.7 Leverage Ratio Distribution of Households in %	35
1.8 Non-targeted Inequality Measure	36
1.9 Variables relative to the median, ordered by net wealth	36
1.10 Decomposition of the Equity Premium	37
1.11 Business Cycle Statistics	46
1.12 Median % Asset Holdings Change in a Crisis	49
1.13 Median % Consumption Change	49
1.14 Comparison of Dynamics during Sudden Stops	50
1.15 Sudden Stop Deviations: Different Heterogeneous Economies	51
1.16 Median % Asset Holdings Change in a Crisis with Dividend Tax	55
1.17 Median % Consumption Change with Dividend Tax	56

FDI Flows and Sudden Stops in Small Open Economies	66
2.1 Calibrated Parameters	90
2.2 Simulated Statistics	93
2.3 Descriptive Regression	95
2.4 List of Countries	106
2.5 List of Sudden Stops	107
 <i>FiPIt</i>: A Simple, Fast Global Method for Solving Models with Two	
Endogenous States & Occasionally Binding Constraints	108
3.1 Calibrated Parameter Values	135
3.2 Long-run Moments & Performance Metrics: Sudden Stops Model ($\kappa =$ 0.2)	138
3.3 Absolute Values of Differences in Equilibrium Functions Relative to <i>FiPIt</i> Solution	139
3.4 Amplification and Asymmetry of Sudden Stop events	145
3.5 Sudden Stops Model Variations: Working Capital & Discounting . . .	149
3.6 Sudden Stops Model Variations: Collateral Coefficient	150
3.7 Sudden Stops Model Variations: Labor Elasticity & Risk Aversion . .	151
3.8 Files Included in the <i>MendozaVillalvazoFiPItCode</i> Directory	156
3.9 Parameter Values	160

LIST OF ILLUSTRATIONS

Inequality and Asset Prices during Sudden Stops	1
1.1 Household Distribution of Annualized % Change in Consumption . . .	14
1.2 Negative Leverage Ratio Histogram	15
1.3 Consumption Gini Coefficient	16
1.4 Joint leverage ratio and debt-to-expenditure ratio distribution	38
1.5 Stationary bond and asset policies as a function of current asset holdings	40
1.6 Stationary bond and asset policies as a function of current bond holdings	41
1.7 Effect of Non-insurable Individual Shocks in the Bond Policy	43
1.8 Effect of Non-insurable Individual Shocks in the Asset Policy	43
1.9 Event Study of a Sudden Stop	47
1.10 Severity of Sudden Stops and Inequality	51
1.11 Event Study of a Sudden Stop in Simulated Economies	52
1.12 Net Foreign Asset Position Event Study of a Sudden Stop in Simulated Economies	53
1.13 Stationary Bond Policies without and with a Dividend Income Tax .	55
1.14 Quantities and Consumption determinants	63
1.15 Asset Prices	64
FDI Flows and Sudden Stops in Small Open Economies	66

2.1	Number of Sudden Stops by year and by classification of economies. .	67
2.2	Financial Account in Emerging Economies	74
2.3	Financial Account in Advanced Economies	75
2.4	Event Study of a Sudden Stop	78
2.5	US real interest rate and realized volatility	81
2.6	Simulated Event Study of a Sudden Stop	94
2.7	Impulse response analysis after a permanent increase in the interna- tional interest rate volatility	97
2.8	Impulse response analysis after a temporary increase in the interna- tional interest rate volatility	98
2.9	Impulse response analysis after a permanent increase in the risk of expropriation	99
2.10	Impulse response analysis after a temporary increase in the risk of expropriation	100
2.11	Episodes of Expropriations	100

<i>FiPIt</i>: A Simple, Fast Global Method for Solving Models with Two Endogenous States & Occasionally Binding Constraints	108
3.1 Long-run Distributions of the Sudden Stops Model Solved with FiPIt	141
3.2 Equilibrium Recursive Functions of the Sudden Stops & RBC Models	143

Chapter 1

Inequality and Asset Prices during Sudden Stops

BY SERGIO VILLALVAZO MARTÍN[†]

1.1 Introduction

In the last 30 years, 58 financial crises have occurred in both emerging and developed economies of the Sudden Stop type, each characterized by episodes of a large reversal in the current account deficit.¹ The occurrence of these crises has led to a vast literature that studies Sudden Stops using models with financial frictions but assuming a representative-agent framework. In the paper in this chapter, we argue that inequality in wealth and leverage across households plays an important role in determining the aggregate effects of a financial crisis.² Specifically, an economy's aggregate ex-

[†]University of Pennsylvania.

¹See Bianchi and Mendoza (2020) for a recent survey and review of the stylized facts of Sudden Stops.

²Figure 1.10 shows descriptive evidence that emerging economies are more unequal than advanced economies, and that Sudden Stop episodes are more severe in more unequal economies.

posure to tighter financial conditions depends on the share of financially vulnerable households defined as those that end up constrained when the crisis happens. Sudden Stops are characterized by large declines in asset prices, which affect households differently depending on their balance sheet. For example, micro-data evidence from Mexico (an open economy commonly used to study Sudden Stops) shows that during the 2009 crisis, households with high leverage decreased their expenditures by 6.2% while non-leveraged households increased their expenditures by 5.4%. Moreover, the value of asset holdings of wealthy households with low leverage increased 64.6% while wealthy households with high leverage fire-sold and decreased the most their assets during the crisis. Hence, studying only aggregate dynamics misses the fact that financial crises do not affect all households in the same way and that inequality has aggregate implications.

This paper addresses this issue by examining the cross-sectional dimension of the debt-deflation mechanism introduced by Fisher (1933). This mechanism works as follows. After a *negative*³ aggregate shock that tightens the financial conditions of the economy, financially constrained agents sell part of their collateralizable assets, which puts downward pressure on asset prices. As asset prices drop, (possibly more) financially constrained agents have to sell a larger asset position, which causes feedback that puts additional downward pressure on asset prices, and this, in turn, further tightens aggregate financial conditions. This paper posits that the cross-sectional dimension of the debt-deflation mechanism matters for macro dynamics of Sudden Stops via two opposing effects: First, a crisis-dampening effect that weakens the debt-deflation mechanism because unconstrained wealthy households can buy the depressed

³Commonly studied negative shocks in small open economy models are an increase in the international interest rate, a decrease in total factor productivity, a drop in the terms of trade, or an ad-hoc tightening of the financial conditions of the economy. In this paper, the financial tightening shock will be a hike in the international interest rate.

assets fire-sold by financially constrained households. Second, a crisis-amplifying effect that strengthens the debt-deflation mechanism because of financially vulnerable households that become credit-constrained as asset prices fall. As aggregate financial conditions tighten, such households also have to sell assets, increasing the downward pressure on asset prices. Because these two cross-sectional effects constitute opposing forces, the role of the cross-section and inequality during crises is quantitatively ambiguous. Hence, this paper conducts a quantitative investigation of the degree to which the severity of Sudden Stops crises is affected by inequality in an economy.

To shed light on the empirical relevance of these issues, we examine a panel household survey for Mexico that provides evidence of the dampening and amplifying cross-sectional effects. Moreover, we test – and reject – the individual complete-market hypothesis. These results support our decision to use a heterogeneous-agent framework to study financial crises and cross-sectional dynamics in households’ consumption and portfolio choice

Then, the paper conducts a quantitative analysis of the effect of wealth inequality on Sudden Stops. To this end, we propose a small-open-economy, asset-pricing Bewley model with debt and assets, an endogenous occasionally-binding loan-to-value (LtV) collateral constraint, and aggregate risk. At the individual level, markets are incomplete, and households face both idiosyncratic labor and dividend income risk. The combination of the dividend risk with an imperfect debt market (the LtV constraint) generates an *asset-wealth trade-off*: more asset holdings relax the collateral constraint and allow for better consumption smoothing (reducing consumption volatility) but also, more asset holdings increase the dividend risk exposure which leads to higher income volatility of the household (increasing consumption volatility), incentivizing additional precautionary savings. This trade-off makes high-dividend asset-rich households deleverage faster than low-dividend households, producing an

empirically plausible leverage ratio distribution with wealthy unconstrained households that face non-degenerate portfolio choices.

In a version of the model calibrated to an emerging economy (Mexico), the quantitative analysis shows that the dampening effect dominates and asset prices drop less in heterogeneous-agents economies. In contrast to the representative-agent framework, the model produces an empirically plausible leverage ratio distribution and generates persistent current account reversals with larger drops in consumption driven by the most leveraged households, consistent with the data. Moreover, calibrating the model to an advanced economy where the dividend risk is one-half of the benchmark emerging-markets model, the average net foreign debt position is twice as large, consumption drops 0.8 percentage points less, and asset prices drop 0.4 percentage points less. Hence, the model predicts that in economies with lower dividend return volatility, income inequality is lower, the economy supports larger debt positions and Sudden Stop crises are less severe, as observed in the data.

The analysis also shows that a constant 50% tax on dividend returns designed to lower income inequality generates more frequent but less severe crises. In particular, the probability of a Sudden Stop increases from 2.3% to 2.5%, and the current account relative to GDP reversal is 0.9 percentage point smaller. The intuition for this result comes from an equilibrium effect on asset prices. Under a redistributive dividend tax, two things happen. First, households have a less potent precautionary savings motive because they are effectively less exposed to dividend risk. Consequently, they demand fewer bonds (or more debt if the bond holdings are negative) and less domestic assets. Hence, the domestic asset's equilibrium price drops to clear the market. On average, the asset price is 54% smaller because of the dividend tax. Since the smaller asset's price tightens the debt limit for every household (the pecuniary externality), the long-run share of financially constrained households increases from 3.4% to 8.2%. This

effect increases the economy’s exposure to changes in the international interest rate and generates more frequent crises. Nonetheless, the second effect of the redistributive dividend tax generates less severe crises in terms of the current account reversal, and aggregate consumption drops 1 percentage point less. Since the financially vulnerable households have effectively less debt because of the smaller asset price that tightens the debt limit, their international bond adjustment is smaller and, together with the redistributive government transfers, the drop in every household’s consumption, but especially the high leveraged, is smaller.

After reviewing the literature in Section 1.2, in Section 1.3 we describe the empirical evidence that supports the cross-sectional effects of the debt-deflation mechanism. The proposed model is described in Section 1.4. Section 1.5 describes the cross-sectional effects through the lens of the model. Section 1.6 presents the quantitative analysis and Section 1.7 concludes.

1.2 Related Literature

This paper contributes to three strands in the economics literature. In the first strand, Sudden Stop crises with financial frictions have been studied using representative-agent models. For instance, Mendoza (2010) studies Sudden Stops in a standard representative firm-agent real business cycle model augmented with a debt-deflation mechanism. He introduces a loan-to-value collateral constraint that generates a pecuniary externality, reflecting that agents do not internalize how their decisions today affect the equilibrium *Tobin’s Q* price of capital that tightens or loosens the debt capacity. In a related paper, Mendoza and K. A. Smith (2006) study the debt-deflation mechanism in a small open economy with a representative agent that trades domestic equity with a foreign investor. In their model, the combination of a collateral

constraint and equity trading costs can produce realistic Sudden Stops. Our paper complements both studies, yet it differs fundamentally from them because we study the cross-sectional dimension of the debt-deflation mechanism. To this end, we introduce market incompleteness at the individual level and study how the distribution of households along bonds, assets, and individual productivities affects the asset's price, portfolio choices, and consumption dynamics during crises.

A second strand of the literature focuses on asset prices in closed economies with individual incomplete markets. Aiyagari and Gertler (1991) study asset prices and particularly the equity premium puzzle (see Mehra and Prescott, 1985) in a closed economy with two assets (bonds and stocks), adjustment costs, and individual labor income risk. The authors conclude that the difference in relative adjustment costs between assets and the need to trade assets for consumption smoothing – introduced by the individual market incompleteness – can generate a spread between the return on bonds and stocks. Heaton and Lucas (1996), who study an economy with two types of agents, income risk, adjustment costs, short-sales constraints, and debt constraints, find that the adjustment costs can generate higher equity premiums. Studying the excess volatility in asset prices that a loan-to-value constraint causes, Aiyagari and Gertler (1999) explain price volatility in a model with limited heterogeneity. In their environment there are only two representative agents: a household and a trader, and when the trader is constrained, the multiplier in the collateral constraint is active for the whole population of traders. This translates into higher volatility in asset prices. More recently, Storesletten, Telmer, and Yaron (2007) show that in a life-cycle model, the effects of idiosyncratic labor risk are quantitatively significant if the idiosyncratic risk becomes more volatile during economic contractions. They further demonstrate that idiosyncratic risk inhibits inter-generational risk sharing, imposing a disproportionate share of aggregate risk on the wealthy middle-aged cohorts who

demand an equity premium for their exposure to this risk. In their setting, the young cohorts do not hold equity to avoid the counter-cyclical volatility risk. Our paper differs from these because we model a small-open-economy with a continuum of agents. This allows analyzing the distributional effects of an endogenous occasionally-binding constraint that introduces a pecuniary externality. Moreover, we show that in our setting, the equity premium can be decomposed into a constraint effect, a risk effect, a trading cost effect that is expected to be close to zero, and a short-sales effect. In fact, the trading cost effect will only be non-zero because of the combination of the collateral constraint and the trading cost function. Hence, most of the risk compensation proceeds from the LtV constraint and individual risk.

A third strand studies the macroeconomy accounting for individual heterogeneity, a line of inquiry begun with the pioneering work of Krusell and A. A. Smith (1997), who developed quantitative tools to analyze economies in which the market clearing price is a function of the distribution of agents (and not only of the mean aggregate state) with individual incomplete markets and aggregate risk. Mendoza, Quadrini, and Rios-Rull (2009) examine how global imbalances can be precipitated by the integration of economies that have different financial markets development. They study the transition path after an unexpected integration of economies and analyze the global balance sheet and equilibrium interest rates. In a related paper, Kaplan and Violante (2014) study households with access to two types of assets that differ in their liquidity. Guerrieri and Lorenzoni (2017) study the transition path in a closed economy that experienced an unexpected tightening in the exogenous debt limit. Finally, in a recent working paper, Huo and Ríos-Rull (2016) examine the effect of asset prices in a closed economy without aggregate risk and study the transition after an unexpected shock in the financial conditions. In contrast, we study the general equilibrium in a small open economy with aggregate risk and individual labor

and dividend productivities. This setup, augmented with an individual loan-to-value collateral constraint, allows us to analyze the cross-sectional dimension of the debt-deflation mechanism and the pecuniary externality that it generates. Finally, in a series of recent empirical papers that study the relationship between income inequality and crises, Bordo and Meissner (2012) and Morelli and Atkinson (2015) study the predictive power of rising income inequality on financial crises without finding conclusive evidence. One exception is Kumhof, Rancière, and Winant (2015), who propose a model to study the effect of changes in the top income distribution on household leverage and crises. Lastly, Guntin, Ottonello, and Perez (2020) use micro-data to assess individual consumption changes in episodes of large aggregate consumption adjustments. The authors argue that consistent with the permanent income hypothesis, households with high income and liquid assets adjust their consumption severely during such episodes. The present paper complements but differs fundamentally from these papers because it studies a model with ex-ante homogeneous agents with ex-post heterogeneity and uses this heterogeneous agent framework to study Sudden Stops and the cross-sectional dynamics in the consumption and portfolio choice of households. Moreover, we document the importance of leverage and not only the liquidity of assets. In particular, we find that during a Sudden Stop, households with high leverage adjust the most their consumption.

1.3 The Cross-Sectional Effects in the Data

This section first describes the data used to show that the cross-sectional effects of the debt-deflation mechanism are empirically relevant. Then, sorting the households according to their net wealth and leverage ratio, we obtain the changes in their individual asset values and consumption during the 2009 Sudden Stop crisis. The

results show that the households in the top decile of wealth and top decile of leverage ratio fire-sold the most their assets while the low-leveraged households increased their asset holdings.

1.3.1 Description of the data

We use data from The Mexican Family Life Survey (MxFLS) for the three available waves: 2002, 2005, and 2009. The MxFLS is a longitudinal household survey that collected information from a representative sample of approximately 8,400 households in 150 localities throughout Mexico. The survey covers information on expenditures, income, assets, and liabilities.⁴ The MxFLS is representative at the national, urban-rural, and regional level.⁵ The sample selection criterion we used corresponds to the households that answered the survey in all three waves. The resulting sub-sample corresponds to 78% of the households in 2005.

Table 1.1 shows the mean net wealth, the portfolio decomposition, and the leverage ratio in 2005 by deciles of the net wealth distribution. The leverage ratio is defined as the household's total debt over the sum of the household's assets. As the second and third rows show, Mexican households' wealth is mostly in physical assets (real estate and other durable goods). Although the proportion of debt decreases as households have higher net wealth, as we can see from the last two rows of the Table, there are leveraged and non-leveraged households in each of the deciles. The next section will analyze the asset and consumption dynamics for households grouped by their level of leverage ratio and net wealth.

⁴To the best of our knowledge, this survey is the only publicly available data source that covers information about the households' stock of assets and liabilities.

⁵For a detailed description of the survey see Rubalcava and Teruel (2006) and Rubalcava and Teruel (2013).

Table 1.1: Mean net wealth and its composition by deciles in 2005

	I	II	III	IV	V	VI	VII	VIII	IX	X
Net Wealth	-796	732	2,507	5346	9,222	14,566	20,697	29,622	45,068	203,451
Real Estate Assets	-62.4%	21.9%	47.1%	69.5%	75.9%	80.9%	82.7%	83.2%	82.3%	74.7%
Other Assets	-85%	89.8%	50.2%	30.8%	23.8%	20%	15.8%	14.2%	14%	10.2%
Financial Assets	-6.4%	9.7%	12%	7.4%	5.2%	4.7%	3.8%	5.1%	6.1%	16.3%
Debt	253.8%	-21.4%	-9.3%	-7.6%	-4.8%	-5.6%	-2.2%	-2.5%	-2.4%	-1.1%
Leverage Ratio										
Mean	0.8	0.05	0	0.01	0.01	0.03	0.01	0	0	0
p90	1.83	0.36	0.18	0.15	0.12	0.09	0.05	0.06	0.05	0.04
p10	0	0	0	0	0	0	0	0	0	0

Notes: Ordered by deciles of net wealth in dollars of 2005. Source: MxFLS.

1.3.2 Stylized Facts: Differentiated Individual Effects

Mexico, as almost any other open economy, experienced a severe Sudden Stop crisis in 2009. Aggregate data shows a current account reversal of 1.5 percentage points relative to GDP, a 7% drop in per capita consumption, and house prices 4% below the pre-crisis trend in 2010 (for an overview of the aggregate time series see Appendix 1.A.1). Moreover, the MxFLS survey shows that from 2005 to 2009, the sum of the households' gross asset values dropped 1%. At the household level, however, the crisis had different effects depending on the composition of their balance sheets.

Supporting evidence of the cross-sectional effects:

The dampening cross-sectional effect comes from the unconstrained wealthy households that can buy the depressed assets fire-sold by the financially constrained households during a crisis. Table 1.2 shows the median change in the real estate owned by households sorted out according to their net wealth and leverage ratio in 2009.⁶ Wealthy households correspond to the top decile of net wealth, and the financially constrained households correspond to the top decile of the leverage ratio. As shown in the Table, the real estate held by wealthy unconstrained households (top right cell) increased by 59.4% while the rest of households experienced drops in their asset holdings. Hence, this evidence supports the dampening effects coming from the cross-sectional dimension: wealthy unconstrained agents take advantage of the depressed prices and increase their asset positions.

Assuming that there were no creation or destruction of real estate, then it must be the case that since the assets held by the unconstrained wealthy agents increased, they were necessarily buying assets from someone else. Hence, other households were selling their assets. Since the amplifying effect comes from the households that are

⁶The survey data corresponds to the value of real estate. To obtain the quantity change, we deflated the value change with the aggregate house price index.

close to becoming financially constrained, and once the mechanism is triggered, they end up financially constrained and strengthen the downward pressure on asset prices. The magnitude of the numbers in the Table suggests that the wealthy financially constrained – the households in deciles X according to the net wealth and to the leverage ratio – fire-sold the most their assets putting downward pressure on their prices. Furthermore, wealthy financially vulnerable – the households in decile X according to the net wealth and decile IX according to the leverage ratio – also ended up fire-selling their assets as the financial conditions tightened. Hence, this evidence supports the amplifying effects coming from the cross-sectional dimension: financially vulnerable agents end up constrained and decrease their asset positions, increasing downward pressure on asset prices.

Table 1.2: Median % Real Estate Change 2005-09

Leverage Ratio	Net Wealth	
	I-IX (Non-Wealthy)	X (Wealthy)
I-VIII (Low-LR)	-1.1	59.4
IX (High-LR)	-1.9	-15.0
X (Very High-LR)	-1.4	-36.5

Notes: Ordered by deciles in 2009. Source: MxFLS.

Additionally, in Table 1.3 we show the median change in the consumption of the households according to their leverage ratio in 2005. During the crisis, households that in 2005 were highly leveraged (bottom row) decreased by 6.2% their consumption. These households were the most affected by the crisis since right before the crisis happened, they were the most exposed to changes in the financial conditions of the economy. In contrast to the declines in consumption of the high leveraged households, the ones in the first decile that mostly have no debt and are net savers, increased their consumption by 5.4%. Households that were moderately leveraged – deciles II to IX

– increased their consumption by less than the non-leveraged households supporting a potential snowball effect: as the financial conditions tightened because financially constrained agents fire-sold their assets, financially vulnerable households ended up constrained. Moreover, these dynamics are different during normal years. In the first column of the Table, we can see that households that end with low leverage ratios are the ones most exposed to idiosyncratic shocks. While the moderately leveraged households, who have debt capacity but are not financially constrained, increased their consumption.

Table 1.3: Median % Consumption Change

	Normal Times	Crisis Times
Leverage Ratio	2002-05	2005-09
I	-8.8	5.4
II-IX	0.2	3.6
X	-1.9	-6.2

Notes: Ordered by deciles in 2005. Source: MxFLS.

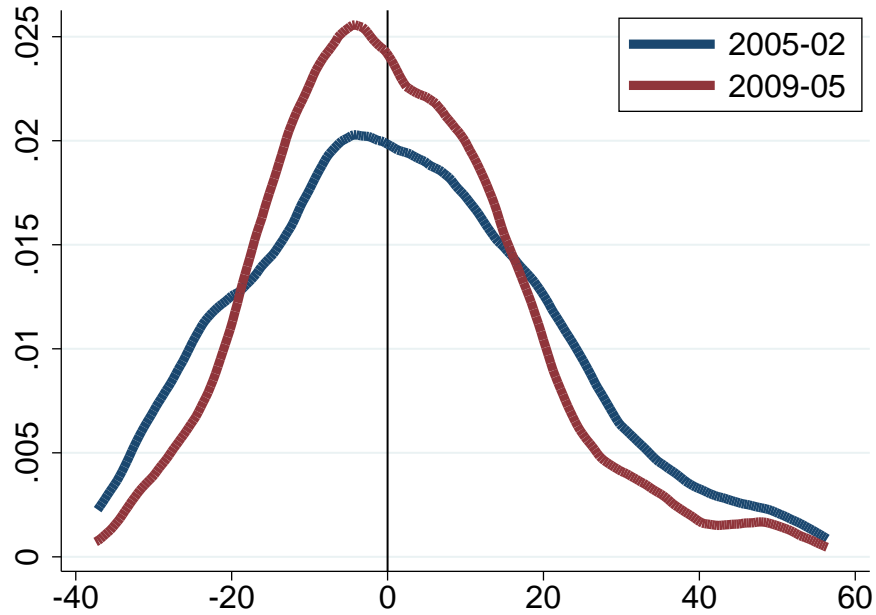
1.3.3 Stylized Facts: Heterogeneous Consumption Dynamics

In this section, we give evidence that the households have heterogeneous consumption dynamics and that the modeling choice of a heterogeneous agent framework is supported by the data. Following Jappelli and Pistaferri (2017), we perform a test of the complete-market hypothesis for Mexico. Under complete markets, changes in individual consumption depend only on aggregate fluctuations common to all individuals. To perform the test, we estimate the following regression

$$\Delta \log c_t^i = \beta \Delta \log C_t + \delta \Delta \log y_t^i + u_t^i \quad (1.1)$$

where c_t^i is the household i consumption in C_t is the aggregate consumption in year t and y_t^i is the household i income in year t . We reject at 1% significance level the joint test of $\beta = 1$ and $\delta = 0$. The point estimates with standard errors in parenthesis are $\beta = 0.73$ (0.22) and $\delta = 0.05$ (0.006). Which are similar to the evidence from Thailand presented in Townsend (1995). Moreover, as we can see in Figure 1.1 changes in consumption vary across households both in normal and crises years. However, during the crisis, there is a larger negative mass and a more concentrated distribution.

Figure 1.1: Household Distribution of Annualized % Change in Consumption



Notes: The distributions are truncated at the top and bottom 1%. Source: MxFLS.

Additionally, Table 1.4 and Figure 1.2 show how the leverage ratio distribution of households changed before and during the crisis. We can see that the mass of financially constrained and the mass of indebted households increased when there was high aggregate liquidity (2002 to 2005). The complement of these changes is that the mass of savers decreased during the same period. This suggests that the economy

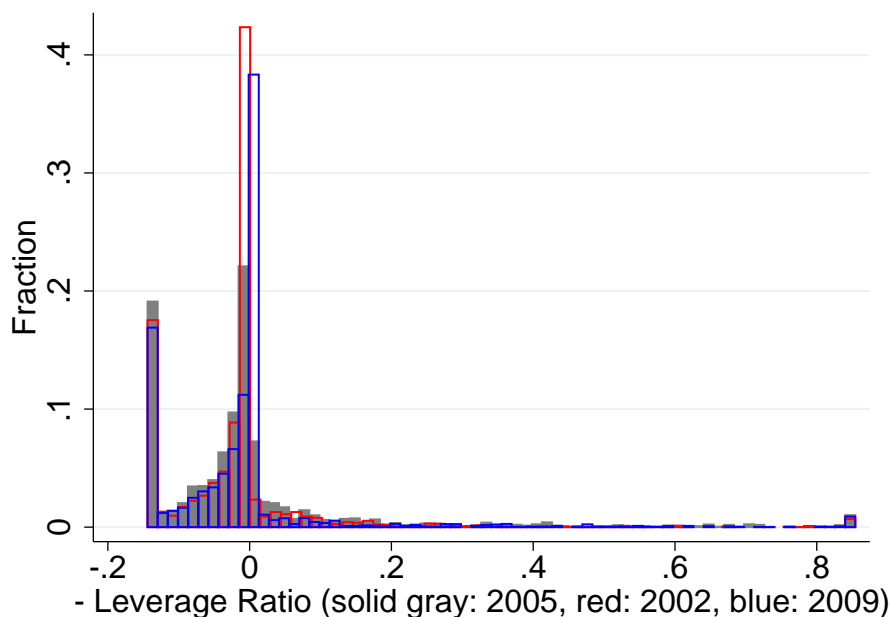
moved to a more exposed aggregate state since more households had positive leverage, and more households were becoming financially constrained. As the crisis unfolds and aggregate liquidity is reduced, households, both financially constrained and indebted, deleveraged, and more became net savers.

Table 1.4: Distribution of Households in %

	2002	2005	2009
Savers (leverage ratio ≤ 0)	37.9	24.6	46.5
Indebted not constrained (leverage ratio $\in (0, 0.144)$)	45.7	57.2	37.5
Indebted constrained (leverage ratio ≥ 0.144)]	16.4	18.2	16.0

Notes: Truncated at a leverage ratio of 14.4%. Source: MxFLS.

Figure 1.2: Negative Leverage Ratio Histogram

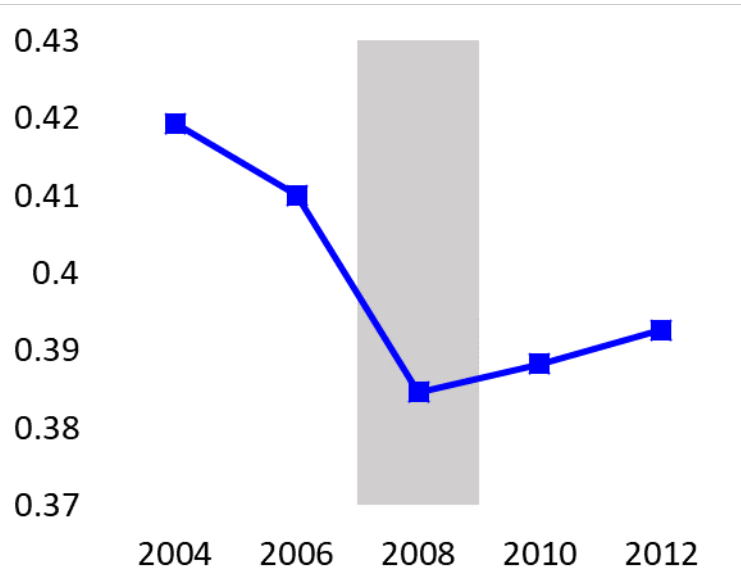


Notes: The leverage ratio corresponds to the total debts over the total assets of the household. Positive values of the negative leverage ratio correspond to households with net savings and negative values correspond to households with net debts. The distribution is truncated at the mean leverage ratio of 0.14. Source: MxFLS.

Finally, we complement the evidence from the MxFLS with the Income and Expenditure Household Survey (ENIGH). This survey is cross-sectional and is done

every two years. In Figure 1.3 we show the Gini coefficient for consumption, and we can see that during the crisis, consumption inequality decreased more than the pre-crisis trend. This evidence is in line with the higher concentration documented in Figure 1.1.

Figure 1.3: Consumption Gini Coefficient



Notes: A larger Gini coefficient means more inequality. Source: ENIGH.

Having documented stylized facts about households' cross-section, we describe the proposed model that accounts for the households' balance sheet heterogeneity in the next section.

1.4 Model

1.4.1 Environment

The model proposed here is a Bewley model of a small open economy with international bonds, domestic equity, and an endogenous occasionally binding constraint. Time is discrete and infinite $t = 0, \dots, \infty$. The economy is populated by a unit mea-

sure of households. There are two financial assets: a one-period risk-free international bond that the households can trade with the rest of the world and a risky domestic asset (land) that is only tradable between the households and is subject to a trading cost.⁷ Borrowing is subject to a loan-to-value (LtV) collateral constraint by which the households' international debt cannot exceed a fraction of the market value of their assets, i.e., the domestic asset is collateralizable.⁸ Regarding the financial market's structure in the economy, markets are incomplete at the aggregate and individual levels. With respect to the aggregate risk, the economy is subject to an aggregate shock that determines the international interest rate. Concerning the individual risk, the households face non-insurable idiosyncratic labor income risk and dividend income risk. The latter risk means that households buy ex-ante identical shares of the risky domestic asset but get ex-post heterogeneity in the return. Evidence of a similar individual return on wealth is documented by Fagereng et al. (2020) and related individual capital income risk has been used by Angeletos (2007), Mendoza, Quadrini, and Rios-Rull (2009), Benhabib, Bisin, and Zhu (2011) and Hubmer, Krusell, and Smith Jr (2020). The combination of the dividend risk with an imperfect debt market (the LtV constraint) generates an *asset-wealth trade-off*: more asset holdings

⁷The assumption of only domestic trading could be relaxed to allow foreign ownership up to a certain percentage of the shares in the economy. With an exogenous stochastic foreign demand for domestic shares, asset prices could become more volatile.

⁸The micro-foundations of the collateral constraint are similar to the ones presented by Bianchi and Mendoza (2018) extended for an economy with non-insurable idiosyncratic risk. Specifically, the LtV constraint is derived from an incentive compatibility constraint resulting from a limited enforcement problem. In an economy where debt contracts are signed with creditors in a competitive environment and households can always switch to another creditor at any point in time. At the beginning of the period credit and asset markets open, production happens and households choose b_{t+1}^i with price R_t^{-1} and a_{t+1}^i with price q_t . Then, markets close, and households decide to divert the resources from the credit and default. Local competitive financial intermediaries monitor costlessly who diverts resources and seize a fraction κ of the household asset holdings, which are $q_t a_{t+1}^i$. After defaulting, the household regains access to credit markets instantaneously and repurchases the assets that investors sell in open markets at a price q_t . In this environment, a household that borrows $-R_t^{-1} b_{t+1}^i$ and engages in diversion activities gains $-R_t^{-1} b_{t+1}^i$ and loses $\kappa q_t a_{t+1}^i$. Hence, households repay if and only $-R_t^{-1} b_{t+1}^i \leq \kappa q_t a_{t+1}^i$.

relax the collateral constraint and allow for better consumption smoothing (reducing consumption volatility) but also, more asset holdings increase the divided risk exposure which leads to higher income volatility of the household (increasing consumption volatility), incentivizing additional precautionary savings. This *asset-wealth trade-off* will be studied in Section 1.5.1.

1.4.2 Households

There is a continuum unit measure of households. Each household $i \in [0, 1]$ maximizes:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^i) \right], \quad (1.2)$$

where c_t^i is consumption of household i , $\beta \in (0, 1)$ is the common discount factor and the utility function, $u(\cdot)$, has a common constant relative risk aversion (CRRA) form. Households have access to the international bond market and the domestic asset market. However, since debt markets are imperfect, only secured-debt is available: household assets serve as collateral. At the beginning of the period, each household holds b_t^i risk-free international bonds, a_t^i shares of the risky domestic asset that has an endogenous price q_t and pays a dividend d_t^i . The household receives labor endowment income w_t^i and uses funds to buy consumption goods c_t^i , bonds to carry for the next period at an exogenous price equal to the inverse of the gross international rate R_t and asset holdings to carry for next period facing a quadratic trading cost of the form $\Phi(a_{t+1}^i, a_t^i) = \frac{\phi}{2}(a_{t+1}^i - a_t^i)^2$. This cost reflects that trading the domestic asset requires a higher level of financial knowledge relative to the bond market and that physical assets are relatively less liquid than bonds. The household's budget constraint is

$$c_t^i + R_t^{-1}b_{t+1}^i + q_t(a_{t+1}^i + \Phi(a_{t+1}^i, a_t^i)) = w_t^i + a_t^i(q_t + d_t^i) + b_t^i \quad (1.3)$$

Households face a loan-to-value constraint that limits their ability to leverage foreign debt on domestic asset holdings. Next period debt (negative bonds) can not exceed a constant fraction κ of the market value of asset holdings. The collateral constraint is

$$R_t^{-1}b_{t+1}^i \geq -\kappa q_t a_{t+1}^i. \quad (1.4)$$

In addition, there is a short-sales constraint on the asset $a_{t+1}^i \geq 0$.⁹ Note that the portfolio choice problem is well defined given the combination of the trading costs in the asset market and the loan-to-value debt constraint.

Lastly, the income of the households is composed of an idiosyncratic and an aggregate part like in Benhabib, Bisin, and Zhu (2015). The individual wage takes the form $w_t^i = \epsilon_t^{i,w} w$ and the individual rate of return $d_t^i = \epsilon_t^{i,d} d$. Where $\{\epsilon_t^{i,w}, \epsilon_t^{i,d}\}$ correspond to the idiosyncratic risk components which will be specified in the next section, and $\{w, d\}$ correspond to the aggregate, exogenous, and constant components.¹⁰

1.4.3 Exogenous stochastic processes

The economy is exposed to only one aggregate shock. The process for the international interest rate is $R_t = \epsilon_t^R \bar{R}$ and $\log(\epsilon_t^R) = \rho_R \log(\epsilon_{t-1}^R) + \eta_t^R$ with $\eta_t^R \sim \mathcal{N}(0, \sigma_R^2)$. Regarding the individual shocks, the individual wage takes the form $w_t^i = \epsilon_t^{i,w} w$ and $\log(\epsilon_t^{i,w}) = \rho_w \log(\epsilon_{t-1}^{i,w}) + \eta_t^{i,w}$ with $\eta_t^{i,w} \sim \mathcal{N}(0, \sigma_w^2)$, and the individual dividend takes the form $d_t^i = \epsilon_t^{i,d} d$ and $\log(\epsilon_t^{i,d}) = \rho_d \log(\epsilon_{t-1}^{i,d}) + \eta_t^{i,d}$ with $\eta_t^{i,d} \sim \mathcal{N}(0, \sigma_d^2)$. Note

⁹The short-sales constraint is needed to ensure that the state space of asset holdings is compact and that the LtV constraint is not irrelevant. If unlimited short selling of assets were possible, households could always undo the effect of Equation 1.4.

¹⁰The structure of the income endowments is similar to an economy in which households supply 1 unit of labor inelastically, and production is done with a competitive constant returns to scale production function that only demands aggregate labor, and pays competitive wages w to each household. Additionally, households have an “ Ak ” production function that uses their individual assets to produce and households obtain dividends d from such production. In the end, households supply effective units of labor and assets so returns are multiplied by the idiosyncratic shocks.

that the idiosyncratic labor and dividend risk that the households face do not have aggregate implications on the returns:

$$\int_0^1 d_t^i di = \int_0^1 \epsilon_t^{i,d} d di = d \quad \text{and} \quad \int_0^1 w_t^i di = \int_0^1 \epsilon_t^{i,w} w di = w.$$

1.4.4 Closing the domestic asset market

The domestic asset is in positive fix net supply equal to \bar{K} and in equilibrium it must be equal to the total asset holdings (demand) of the households. Hence, market-clearing in the asset market requires: $\int_0^1 a_t^i di = \bar{K}$ for every t .

1.4.5 Recursive Formulation

To characterize the problem of the agents and the equilibrium in recursive form we start by defining the states of the economy. Households are heterogeneous in their current holding of bonds, assets, idiosyncratic labor and dividend productivity. The individual states are: $(b, a, \epsilon^w, \epsilon^d)$. We need to keep track of both the individual bonds and assets given the asset trading costs and the imperfect debt market. Let $\Omega(b, a, \epsilon^w, \epsilon^d)$ be the endogenous distribution of households according to their bonds, assets and individual productivities. Regarding aggregate states, to forecast asset prices, the households need to know the distribution of wealth. Hence, the aggregate states correspond to the endogenous distribution Ω , and the exogenous shock to the international interest rate ϵ^R . Letting the superscript $/$ correspond to the variables in

the next period, the recursive problem of a household becomes:

$$\begin{aligned}
v(\underbrace{b, a, \epsilon^w, \epsilon^d}_{\text{Ind. states}}, \underbrace{\Omega, \epsilon^R}_{\text{Agg. states}}) &= \max_{\{c, b', a' \geq 0\}} u(c) + \beta \mathbb{E}[v(b', a', \epsilon^{w'}, \epsilon^{d'}, \Omega', \epsilon^{R'})] \\
&s.t. \\
c + R(\epsilon^R)^{-1}b' + q(\Omega, \epsilon^R)(a' + \Phi(a', a)) &= \epsilon^w w + a(q(\Omega, \epsilon^R) + \epsilon^d d) + b, \quad \text{with mult. } \lambda \\
R(\epsilon^R)^{-1}b' &\geq -\kappa q(\Omega, \epsilon^R)a', \quad \text{with mult. } \mu \\
\Phi(a', a) &= \frac{\phi}{2}(a' - a)^2 \\
\Omega' &= H^\Omega(\Omega, \epsilon^R), \tag{1.5}
\end{aligned}$$

where $H^\Omega(\cdot)$ corresponds to the aggregate law of motion of the distribution of households.

Definition of a Recursive Competitive Equilibrium

Let the individual bond and asset holdings be elements $(b, a) \in [b, \bar{b}] \times [0, \bar{a}] \equiv \mathcal{S}$ and the individual productivities be elements $(\epsilon^w, \epsilon^d) \in \{\epsilon_1^w, \dots, \epsilon_{N_w}^w\} \times \{\epsilon_1^d, \dots, \epsilon_{N_d}^d\} \equiv \mathcal{E}^I$. Let \mathcal{M} be the set of probability measures of the set $\mathcal{S} \times \mathcal{E}^I$ and the aggregate shocks be elements $\epsilon^R \in \{\epsilon_1^R, \dots, \epsilon_{N_R}^R\} \equiv \mathcal{E}^A$. Finally, let the function $\pi(\epsilon'|\epsilon)$ be the exogenous Markov transition probability of next period shocks take the realization ϵ' conditional on the shocks in the current period being ϵ , where $\epsilon = (\epsilon^w, \epsilon^d, \epsilon^R) \in \mathcal{E}^I \times \mathcal{E}^A$. Now we can define a recursive competitive equilibrium.

Definition 1. *A recursive competitive equilibrium in this economy is given by a value function $v : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$, policy functions for the household $c : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$, $b' : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$ and $a' : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$, domestic asset pricing function $q : \mathcal{M} \times \mathcal{E}^A \rightarrow \mathbb{R}$, and an aggregate law of motion $H^\Omega : \mathcal{M} \times \mathcal{E}^A \rightarrow \mathcal{M}$ such that:*

1. Given the asset pricing function and the aggregate law of motion, the value function v satisfies the household's Bellman Equation 1.5 and c , a' , b' are the associated policy functions,

2. For all $\Omega \in \mathcal{M}$ and all $\epsilon^R \in \mathcal{E}^A$, the asset market clears:

$$\int_{\mathcal{S} \times \mathcal{E}^I} a \, d\Omega = \int_{\mathcal{S} \times \mathcal{E}^I} a'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, d\Omega = \bar{K},$$

3. For all $\Omega \in \mathcal{M}$ and $\epsilon^R \in \mathcal{E}^A$, the aggregate resource constraint is satisfied:

$$\begin{aligned} & \int_{\mathcal{S} \times \mathcal{E}^I} c(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, d\Omega + R(\epsilon^R)^{-1} \int_{\mathcal{S} \times \mathcal{E}^I} b'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, d\Omega \\ & + q(\Omega, \epsilon^R) \int_{\mathcal{S} \times \mathcal{E}^I} \Phi(a'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R), a) \, d\Omega = w + \int_{\mathcal{S} \times \mathcal{E}^I} a \epsilon^d \, d\Omega + \int_{\mathcal{S} \times \mathcal{E}^I} b \, d\Omega, \end{aligned}$$

4. The aggregate law of motion is generated by the exogenous Markov process π and the policy functions b' and a' as described below:

Let $(\epsilon^w, \epsilon^d) = \epsilon^I$ and $\epsilon^R = \epsilon^A$ and define the transition function $Q_{\Omega, \epsilon^A} : \mathcal{S} \times \mathcal{E}^I \times \mathcal{B}(\mathcal{S}) \times \mathcal{B}(\mathcal{E}^I) \rightarrow [0, 1]$, where $\mathcal{B}(\cdot)$ is the corresponding Borel set, by

$$Q_{\Omega, \epsilon^A}(b, a, \epsilon^I, \mathcal{S}, \mathcal{E}^I) = \begin{cases} \sum_{\epsilon^{I'} \in \mathcal{E}^I, \epsilon^{A'} \in \mathcal{E}^A} \pi(\epsilon^{I'}, \epsilon^{A'} | \epsilon^I, \epsilon^A), & \text{if } (b'(b, a, \epsilon^I, \Omega, \epsilon^A), a'(b, a, \epsilon^I, \Omega, \epsilon^A)) \in \mathcal{S} \\ 0, & \text{otherwise.} \end{cases}$$

Then, for any $\mathcal{S} \in \mathcal{B}(\mathcal{S})$ and any $\mathcal{E}^I \in \mathcal{B}(\mathcal{E}^I)$ the aggregate law of motion is given by

$$\Omega'(\mathcal{S}, \mathcal{E}^I) = (H^\Omega(\Omega, \epsilon^A))(\mathcal{S}, \mathcal{E}^I) = \int_{\mathcal{S} \times \mathcal{E}^I} Q_{\Omega, \epsilon^A}(b, a, \epsilon^I, \mathcal{S}, \mathcal{E}^I) \, d\Omega.$$

1.5 The Cross-Sectional Effects in the Model

In this section, we study the cross-sectional effects on the credit and equity channel of the economy.

1.5.1 Market Incompleteness and Risk Exposure

The households are exposed to two sources of non-insurable idiosyncratic risk that have different equilibrium implications. Note that the standard Bewley non-insurable persistent labor income risk ϵ^w , together with the constant aggregate labor income endowment assumption implies a fixed labor risk exposure. This means that the exposure to the labor earnings risk is independent of the households' decisions. In contrast, the idiosyncratic persistent dividend productivity, ϵ^d , allows the households to change future risk exposure by changing the next period holdings of the asset.

This varying dividend risk exposure, combined with the loan-to-value collateral constraint, generates an *asset-wealth trade-off*. To see this, first, note that when households are in an adverse state, they can smooth consumption in two ways: by lowering their bond holdings b' (if these are already negative, this means borrow more) or by reducing their asset holdings a' . Given the financial frictions in the debt market (see Equation 1.4), to have credit capacity and hence borrow, the household needs first to save and accumulate assets. Note that although the current dividend return is given since the current asset holdings are fixed in the current period (they are an individual state variable), the household chooses how much future exposure to have by choosing the next period asset holdings a' . Because the flow income of the household is given by $FI(a, \epsilon^w, \epsilon^d) = \epsilon^w w + a \epsilon^d d$, with independent idiosyncratic risks its variance is $\mathbb{V}[FI(a, \epsilon^w, \epsilon^d)] = w^2 \sigma_{\epsilon^w}^2 + a^2 d^2 \sigma_{\epsilon^d}^2$ which is a convex function with respect to the asset holdings. This translates into more income volatility for asset-rich

households. This property of the flow income generates the following trade-off from getting more assets:

1. Households get higher debt capacity that allows higher smoothing and reduces consumption volatility since $R(\cdot)^{-1}b'(\cdot) \geq -\kappa q(\cdot)a'(\cdot)$, incentivizing lower precautionary savings.
2. Households get higher future income risk that increases consumption volatility, incentivizing higher precautionary savings.

In equilibrium, indebted asset-poor households increase their debts as they increase their assets, and for households with high dividend returns, when they become asset-rich, they start deleveraging (precautionary saving motives kick in) and some end up being savers due to the increasing income risk.¹¹ This behavior generates unconstrained wealthy households which endogenously have a diversified portfolio: asset-rich households end up holding both positive international bonds and domestic assets.

Similar trade-offs have been studied in the literature but through different mechanisms. Mendoza, Quadrini, and Rios-Rull (2009) find that an individual investment shock (similar to an individual dividend shock) makes agents lower their debt positions as they increase their net wealth. The outcome for asset-rich households is the same but for different reasons. Because we introduce the shock with persistence (theirs is an *iid* shock) the households with a negative dividend shock want to lower their bond position (or increase debts if negative) as the asset position increases. Moreover, in our paper, introducing the LtV constraint and the individual non-trivial portfolio choice problem makes asset-poor households increase their debts as they increase their assets. In another study, Benhabib, Bisin, and Zhu (2011) show that idiosyncratic

¹¹See the top row of Figure 1.5 in the graphical analysis of the policy functions done for the calibrated stationary model in Section 1.6.2.

capital returns determine the properties of the right tail of the wealth distribution in a Bewley economy. Their theoretical result is in line with the *asset-wealth trade-off* described above since asset-rich households that get a positive dividend shock will increase their net wealth by two sources: by buying more assets and by increasing their bond position (or decreasing their debt if the bond position is negative). Hence the share of wealthy households and the wealth inequality increase. However, again, the combination of the dividend risk with the LtV constraint allows the model to generate an empirical plausible distribution of constrained households, financially vulnerable households that hold debt, and households with positive bond positions (savers).

1.5.2 Financial Premia

In this section, we study the effects that the households' balance sheet heterogeneity introduces. Specifically, we analyze the cross-sectional dimension of the debt-deflation mechanism in terms of the external financing premium and equity premium at the individual and aggregate levels. For simplicity, we omit the state variables and re-introduce the superscript i to identify household-specific variables. Let λ^i , μ^i and ψ^i be the multipliers on the budget constraint, the collateral constraint, and the short-sales constraint, respectively, and let $\tilde{\mu}^i = \frac{\mu^i}{\lambda^i}$ and $\tilde{\psi}^i = \frac{\psi^i}{\lambda^i}$.

Similar to the analysis done by Mendoza and K. A. Smith (2006) but for an economy with heterogeneous agents, from the first-order conditions of household i 's problem we obtain an Euler Equation for individual bonds:

$$\lambda^i R^{-1} - \mu^i R^{-1} = \beta \mathbb{E}[\lambda^{i'}] \quad \Rightarrow$$

$$0 < 1 - \tilde{\mu}^i = \beta R \mathbb{E} \left[\frac{\lambda^{i'}}{\lambda^i} \right] \leq 1 \quad \text{since } \lambda^i > 0, \mu^i \geq 0 \text{ and } \tilde{\mu}^i = \frac{\mu^i}{\lambda^i} \in [0, 1).$$

Let the individual expected effective interest rate be the inverse of the individual

stochastic discount factor $\mathbb{E}[R^{i,eff}] = \mathbb{E}[SDF^i]^{-1} = \mathbb{E}\left[\beta \frac{\lambda^{i'}}{\lambda^i}\right]^{-1}$. Then, from the above Euler Equation we get an individual expected external financing premium on debt:

$$\mathbb{E}[R^{i,eff}] - R = R \frac{\tilde{\mu}^i}{1 - \tilde{\mu}^i} \geq 0. \quad (1.6)$$

This individual premium reflects the fact that when the constraint binds ($\tilde{\mu}^i > 0$), the household would want to borrow more than what the collateral constraint allows. Also, note that it is increasing on $\tilde{\mu}^i$. This means that as the constraint tightens, the household would be willing to pay an interest rate higher than R for more debt.

Similarly, from the first-order conditions of household i 's problem we obtain the Euler Equation for individual assets:

$$q(\lambda^i(1 + \Phi_1^i) - \kappa\mu^i) - \psi^i = \beta\mathbb{E}[\lambda^{i'}(q' + d^{i'} - q'\Phi_2^{i'})],$$

where Φ_j^i corresponds to the partial derivative with respect to argument j . Let $\tilde{d}^{i'} = d^{i'} - q'\Phi_2^{i'}$ and the individual return on the asset be $\tilde{R}^{i,q} = \left(\frac{q' + \tilde{d}^{i'}}{q}\right)$. Then, from the above Euler Equation we get an individual expected equity premium:

$$\mathbb{E}[\tilde{R}^{i,q}] - R = \frac{R \left((1 - \kappa)\tilde{\mu}^i - \text{COV}[SDF^i, \tilde{R}^{i,q}] + \Phi_1^i - \tilde{\psi}^i \right)}{1 - \tilde{\mu}^i}. \quad (1.7)$$

As in Mendoza and K. A. Smith (2006), in Equation 1.7 we see a direct positive effect in the individual equity premium coming from the collateral constraint: as $\tilde{\mu}^i$ increases, the individual equity premium increases by an additive term that multiplies $R(1 - \kappa)$ and by a multiplicative factor $(1/(1 - \tilde{\mu}^i))$ that affects the whole premia. Also, there is a positive risk effect coming from the covariance term that will become more negative due to the precautionary savings.¹² Lastly, there is an ambiguous effect

¹²This risk effect also includes the next period's marginal trading cost effect that is expected to

coming from the marginal trading costs. This last effect is expected to be negative for financially constrained households since when $\tilde{\mu}^i > 0$, the household will sell assets to smooth consumption and $a^{i'} < a^i \Rightarrow \Phi_1^i < 0$. When the constraint binds, a larger equity premium reflects that buying an extra unit of the asset provides an additional benefit since this additional unit also relaxes the constraint. However, this additional benefit is imperfect since κ fraction of the assets is pledgeable as collateral.

The aggregate expected equity rate of return, $\mathbb{E}[R^q]$, can be obtained by first integrating the individual expected asset returns over all the households:

$$\begin{aligned}
\int_0^1 \mathbb{E}[\tilde{R}^{i,q}] di &= \mathbb{E} \left[\int_0^1 \tilde{R}^{i,q} di \right] = \mathbb{E} \left[\int_0^1 \left(\frac{q' + \tilde{d}^{i'}}{q} \right) di \right] = \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} \int_0^1 \tilde{d}^{i'} di \right] = \\
&= \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} \int_0^1 d^{i'} - q' \Phi_2^{i'} di \right] = \\
&= \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} \underbrace{\int_0^1 d^{i'} di}_{=d'=d} + \frac{1}{q} \int_0^1 q' \phi(a^{i'''} - a^{i'}) di \right] = \\
&= \mathbb{E} \left[\frac{q'}{q} + \frac{1}{q} d' + \frac{q' \phi}{q} \left(\underbrace{\int_0^1 a^{i'''} di}_{=\bar{K}} - \underbrace{\int_0^1 a^{i'} di}_{=\bar{K}} \right) \right] = \mathbb{E} \left[\frac{q' + d'}{q} \right] \equiv \mathbb{E}[R^q].
\end{aligned}$$

Then, we use the expected returns derived in Equation 1.7 to obtain a decomposition of the aggregate expected equity premium. Assuming that fraction \bar{I} of households are credit constrained and without loss of generality sorting constrained

increase the precautionary motives. The intuition for this is the following. Note that the household that next period gets a high dividend return will buy more shares, hence $a^{i'''} > a^{i'} \Rightarrow \Phi_2^{i'} < 0 \Rightarrow \tilde{d}^{i','} > d^{i'}$, effectively the individual dividend risk increases due to the trading costs.

households from 0 to \bar{I} we obtain the following result:

$$\begin{aligned}
\mathbb{E}[R^q] - R = & \underbrace{R(1 - \kappa) \int_0^{\bar{I}} \frac{\tilde{\mu}^i}{1 - \tilde{\mu}^i} di}_{\text{Constraint Effect: } +\bar{I} \text{ and } +\tilde{\mu}} \underbrace{- R \int_0^1 \frac{\text{COV}[SDF^i, \tilde{R}^{i,q}]}{1 - \tilde{\mu}^i} di}_{\text{Risk Effect: } "+"} \\
& + \underbrace{R \int_0^1 \frac{\Phi_1^i}{1 - \tilde{\mu}^i} di}_{\text{Trading Cost Effect: } "\approx 0"} \underbrace{- \frac{R}{q} \int_0^1 \frac{\tilde{\psi}^i}{1 - \tilde{\mu}^i} di}_{\text{Short-Sales Effect: } "-"} . \tag{1.8}
\end{aligned}$$

This expression shows that the aggregate excess returns can be decomposed into four effects. First, a positive direct effect coming from the measure of constrained households and from how “strong” the constraint binds. Second, the risk effect coming from the covariance between the individual stochastic discount factor and the individual return on the equity (note that the integral becomes a weighted average of the covariances with larger weights on constrained households since $\tilde{\mu}^i > 0 \Rightarrow 1/(1 - \tilde{\mu}^i) > 1$). Since constrained households are expected to have more negative covariances due to the increased individual consumption volatility and the precautionary savings behavior, we expect a positive risk effect. Third, the trading cost effect, again, the weighted average puts more weight on constrained households, and since $\int_0^1 \Phi_1^i di = 0$ we can expect the aggregate effect to be close to zero and decreasing with respect to ϕ . This trading cost effect comes from the interaction of the collateral constraint and the trading cost function since if there are no constrained households, this term becomes zero. Fourth, a short-sales effect that decreases the equity premium since households with a binding short-sales constraint increase the marginal gain of additional asset holdings and has no effect on the marginal benefit of saving in assets.

Finally, the debt-deflation cross-sectional effects in the risk premium are:

1. Dampening effect: having more unconstrained wealthy households reduces the equity premium by having a smaller risk effect since they are better able to smooth consumption.
2. Amplifying effect: having more financially vulnerable households increases the equity premium due to a larger constraint effect (larger \bar{I}) and by having a larger risk effect since these constrained households have more consumption volatility.

Note that the precautionary behavior introduced by the *asset-wealth trade-off*, under empirically suitable high persistence of the dividend risk, generates unconstrained households. Hence, in the stationary equilibrium the measure of financially constrained households is $\bar{I} < 1$. Intuitively, when households get a high individual dividend return, they accumulate more assets. Since the individual risk is sufficiently persistent, this gives households enough time to become asset-rich and the dividend risk exposure is high enough such that the precautionary savings motive makes households deleverage and become unconstrained. In the next section, we use the model as a measurement device to quantitatively study the cross-sectional effects of a Sudden Stop episode.

1.6 Quantitative Analysis

This section presents the quantitative results of the model. Due to the computational intensity of the solution method, we calibrate the parameters using the stationary model without aggregate risk.¹³ To calibrate the model, we use data for Mexico. Table 1.5 shows the calibrated parameters.

¹³Since the economy has an endogenous occasionally-binding constraint, the household's policy functions are expected to be highly nonlinear, and a global solutions method is needed. We use the *FiPit* algorithm proposed by Mendoza and Villalvazo (2020) to solve the household's problem combined with the stochastic-simulation approach by L. Maliar, S. Maliar, and Valli (2010) and Krusell and A. A. Smith (1997) to solve the aggregate uncertainty problem.

1.6.1 Calibration

Table 1.5: Parameters

Parameter	Value	Source or Target
Calibrated outside of the model		
ν Risk aversion	2	Common in the literature
\bar{R} Interest rate	1.03	Mean interest rate Mexico 1990-2017
κ Debt fraction of collateral	0.14	Mean leverage ratio in 2005
\bar{K} Net asset supply	1	Normalization
Calibrated by simulation		
β Discount factor	0.90	Mean NFA/GDP ratio of -40%
ϕ Trading cost	3.5	Mean transaction cost of 5%
Individual labor income risk		
w Average wage	0.072	See Section 1.6.1
ρ_w Autocorrelation	0.91	
σ_w Std. dev.	20%	
Individual dividend income risk		
d Average dividend yield	0.036	See Section 1.6.1
ρ_d Autocorrelation	0.94	
σ_d Std. dev.	83%	
Aggregate interest rate risk		
R Interest rate value	{1.01, 1.05}	See Section 1.6.1
ρ_R Autocorrelation	0.90	

Regarding the set of parameters that are calibrated outside of the model, we set the household's risk aversion $\nu = 2$ which is a value common in the literature. The average international interest rate equal to 3% which is Mexico's interest rate average between 1990 to 2017. The collateral debt fraction κ equal to 0.14 which is the average leverage ratio in 2005. Lastly, the net asset supply is normalized at 1. Then, we calibrate by simulation the discount factor $\beta = 0.90$ to match the average net foreign asset position relative to GDP for Mexico equal to 40% and the trading cost parameter ϕ equal to 3.5 to obtain an average transaction cost of 5% which is consistent with the estimates from Aiyagari and Gertler ([1999](#)).

To estimate the exogenous earning process we apply the methodology described in Krueger, Mitman, and Perri (2016) using Mexican data.¹⁴ First, we estimate a Mincer log-earnings equation with time fixed effects

$$\log(Y_{a,t}^i) = \beta' X_{a,t}^i + D_t + y_{a,t}^i, \quad (1.9)$$

where each observation corresponds to an individual i , with quarterly age a and in quarter t . $Y_{a,t}^i$ corresponds to the annual income of the person, the vector of controls $X_{a,t}^i$ includes a cubic polynomial on age, dummy variables for the education level and a dummy variable that identifies if the worker is in the informal sector. Finally, D_t corresponds to the time fixed effects dummy variables. After running the regression, we obtain the residuals $y_{a,t}^i$ and assume the income risk follows a stationary process with a persistent and transitory component. The stationarity assumption allows us to drop the time dimension and the income risk model becomes

$$\begin{aligned} y_a^i &= z_a^i + \epsilon_a^i \\ z_a^i &= \rho_w z_{a-1}^i + \eta_a^{i,w} \\ \eta_a^{i,w} &\sim (0, \sigma_w^2), \quad z_0^i \sim (0, \sigma_{z_0}^2), \quad \epsilon_a^i \sim (0, \sigma_\epsilon^2). \end{aligned} \quad (1.10)$$

Now the objective is to estimate the vector of parameters $\theta = (\rho_w, \sigma_w^2, \sigma_{z_0}^2, \sigma_\epsilon^2)$. These

¹⁴There is a vast literature on the estimation of the labor income risk (see Meghir and Pistaferri, 2004, Storesletten, Telmer, and Yaron, 2004, Guvenen, 2007, Heathcote, Storesletten, and Violante, 2010).

parameters are identified with the following theoretical moments:

$$\begin{aligned}
\rho_w &= \frac{\text{COV}[y_a^i, y_{a-2}^i]}{\text{COV}[y_{a-1}^i, y_{a-2}^i]} \\
\sigma_\epsilon^2 &= \text{V}[y_{a-1}^i] - \rho^{-1} \text{COV}[y_a^i, y_{a-1}^i] \\
\sigma_w^2 &= \text{V}[y_{a-1}^i] - \text{COV}[y_a^i, y_{a-2}^i] - \sigma_\epsilon^2 \\
\sigma_{z_0}^2 &= \text{V}[y_0^i] - \sigma_\epsilon^2 .
\end{aligned} \tag{1.11}$$

We use data from the National Survey of Employment and Occupation (ENOE) to do an over-identified GMM estimation with an identity weighting matrix.¹⁵ The ENOE survey is a quarterly household rotating panel with a representative sample of 120,000 households that started in 2005-I. Every household is interviewed for 5 consequently quarters and each quarter 20% of the sample is replaced. As the standard practice in the literature, our sample selection criteria are individuals with ages between 20 and 60, males, and with positive earnings. Table 1.6 shows the estimated parameters and compares them with the literature's estimation done for the US.

We find that the estimated persistence of the income risk process is smaller, and the variance is larger for Mexico compared to the US. A reason for this difference could come from the informal market structure that is common in emerging economies (Leyva and Urrutia, 2020). The Mexican labor market is characterized by having a high informality rate in which more than 50% of informal employment. Since the informal sector is relatively more flexible than the formal sector, it could create a less permanent effect of idiosyncratic shocks. Moreover, Gomes, Iachan, and Santos (2020) find that informality is associated with more volatile earnings. Finally, the combination of a large informal sector and the lack of unemployment insurance could

¹⁵Note that to just-identify the parameters we only need data for ages $(a, a - 1, a - 2)$. Since we are using data for 160 quarterly-ages the system is over-identified.

Table 1.6: Annual Income Process Estimates

	Mexico		US		US	
	Benchmark	Formal Employment	Storesletten, Telmer, and Yaron (2004)	Guvenen (2009)	Krueger, Mitman, and Perri (2016)	
ρ_w	0.906	0.922	0.982	0.988	0.970	
σ_w^2	0.039	0.038	0.024	0.015	0.038	

Notes: The results for Mexico correspond to data from the ENOE from 2005-I to 2014-IV. The estimates are annualized following Krueger, Mitman, and Perri (2016).

also cause a higher income risk.¹⁶ To explore this reason, in the second column, we show the results from the estimation done with a subsample of only formal employment. As expected, the difference narrows, although the change is small. Given that we do not explore specific heterogeneity in the labor markets in the model, we still use as a benchmark the results from the first column that include all the employment. Lastly, the discrete labor income risk process is approximated using a symmetric 2-state Markov chain using a simple persistence rule following Mendoza (2010). The discretized risk takes the values $\epsilon^w \in \{\epsilon_L^w = 0.80, \epsilon_H^w = 1.20\}$ and the probability that the next period realization of the shock is the same as the current period is $Pr[\epsilon^{w'} = \epsilon_j^w | \epsilon^w = \epsilon_j^w] = 0.95$ for $j \in \{L, H\}$.

The dividend income risk plays a key role in the decision rules of the households and drives the *asset-wealth trade-off* discussed in Section 1.5.1. However, a proper estimation of this process is infeasible due to the lack of available data in most economies.¹⁷ Due to the restrictions of the available data for Mexico, we take the following calibration strategy. We jointly calibrate the three parameters that characterize the dividend income risk $(\bar{d}, \rho_d, \sigma_d)$ to match the leverage ratio distribution of households in 2005. Specifically, we focus on three distribution statistics: the measure of savers that have financial assets (negative leverage ratio), indebted households that have positive debts but are not close to their debt limit, and financially constrained households. The calibrated parameters are $(d = 0.036, \rho_d = 0.94, \sigma_d = 0.83)$ and similarly to the labor risk, the discrete dividend risk process is approximated using a symmetric 2-state Markov chain using a simple persistence rule. Hence, the

¹⁶Bosch and Esteban-Pretel (2015) study the consequences on the labor market of implementing an unemployment benefit system in economies with large informal sectors and find that an unemployment benefit could increase the formality rate.

¹⁷One exemption is the work by Fagereng et al. (2020) which estimate the wealth risk using administrative data from Norway and find that there is high heterogeneity in the wealth returns and that these differences are highly persistent.

discretized risk takes the values $\epsilon^d \in \{\epsilon_L^d = 0.17, \epsilon_H^d = 1.83\}$ and the probability that the next period realization of the shock is the same as the current period is $Pr[\epsilon^{d'} = \epsilon_j^d | \epsilon^d = \epsilon_j^d] = 0.97$ for $j \in \{L, H\}$. These estimates imply that the effective dividend yield ($\epsilon^d d$) the households will face can take the following two values: $\{0.6\%, 6.6\%\}$. The matched distribution is shown in Table 1.7. Lastly, the aggregate wage level, w , is set equal to $2d\bar{K}$ such that the average household has a total flow income that correspond to two-thirds labor income and one-third dividend income.

Table 1.7: Leverage Ratio Distribution of Households in %

	Data in 2005	Stationary Model
Savers: leverage ratio ≤ 0	24.6	25.1
Indebted not constrained: leverage ratio $\in (0, 0.14)$	57.2	57.0
Financially constrained: leverage ratio ≥ 0.14	18.2	17.9

Notes: Financially constrained households correspond to the households with leverage ratio above the mean leverage ratio equal to 0.144 in 2005. Source: MxFLS.

The last exogenous process that needs to be calibrated corresponds to the international interest rate. This process will also follow symmetric 2-state Markov chain with values $R \in \{1.01, 1.05\}$ and persistence $\rho_R = 0.90$. These values are common in the literature of small open economies and have been used in studies of the Mexican economy (see Bianchi, 2016).

1.6.2 Stationary Model

In this section, we analyze the stationary equilibrium for an economy in which the interest rate is constant at its steady state value of 3% – i.e., a Bewley economy without aggregate risk. The stationary model does a good job capturing the wealth and consumption inequality, as seen in Table 1.8. This is the result of the *asset-wealth trade-off* described in Section 1.5.1.

Moreover, in Table 1.9 we show the average net wealth, assets, and debts by

Table 1.8: Non-targeted Inequality Measure

	Model	Data
Wealth Gini	0.678	0.733
Consumption Gini	0.305	0.497

Notes: Source: MxFLS.

deciles relative to the median level of each variable for simulated data and observed data in 2005. As we can see in the top and medium rows, the net wealth and assets distributions generated by the model are very close to the ones obtained from the MxFLS in 2005. Regarding the total debt, the only decile that is significantly different is the bottom decile. One possible reason for this difference is that we do not allow the households to default in the model and cannot hold more debt than the collateral limit. Where in the real data, households in the bottom decile have negative net wealth. However, for the rest of the deciles, the model does a good job of capturing the inequality in terms of the net wealth, total assets, and debt.

Table 1.9: Variables relative to the median, ordered by net wealth

	I	II	III	IV	V	VI	VII	VIII	IX	X
Net Wealth relative to median										
Data	-0.1	0.1	0.3	0.6	1	1.6	2.2	3.2	4.9	22.1
Model	0	0.1	0.4	0.7	1	1.5	2.2	3.3	5.8	16.5
Assets relative to median										
Data	0.1	0.1	0.3	0.6	1	1.6	2.2	3.1	4.8	21.3
Model	0	0.1	0.3	0.6	1	1.5	2.2	3.4	5.9	17
Debt relative to median										
Data	4.3	0.4	0.5	0.9	1	1.8	1	1.6	2.4	5.2
Model	0.1	0.2	0.3	0.5	1	1.5	2.3	3.5	6.2	9.5

Notes: Deciles ordered by the net wealth.

Regarding the aggregate equity premium, in Table 1.10 we show its level and decomposition. As expected, the risk component contributes the most to the equity premium, about 60%. The other 40% corresponds to the constraint effect. Note that

the calibration was done to capture the measure of constrained households in 2005 equal to 18% (see Table 1.7). Hence, even if only these households have an active debt constraint, there is an important contribution to the equity premium.

Table 1.10: Decomposition of the Equity Premium

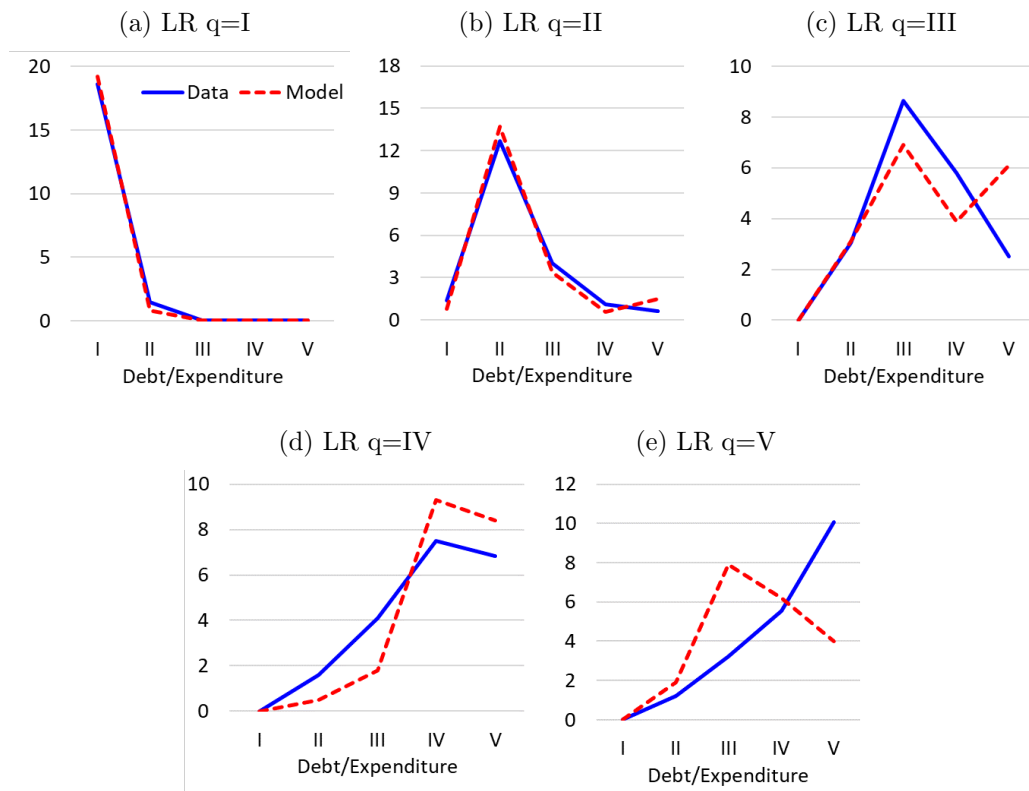
	Model	Data
Equity Premium	4.9%	6.5%
Constraint Effect	39.1%	-
Risk Effect	59.7%	-
Trading Cost Effect	2.7%	-
Short-Sales Effect	-1.5%	-

Notes: Data from Damodaran (2013) corresponds to Mexico in 2005.

Finally, notice that the debt-deflation mechanism affects a household's consumption when two things happen. First, the household must be highly leveraged, so when the collateral constraint tightens, they are close to (or at) the binding region and they need to adjust their asset holdings; and second, the household must have a large debt-to-expenditure ratio so when they have to deleverage, there is a significant impact on their consumption. As a model validation exercise, the following figures show how well the model replicates the distribution of households with respect to the joint leverage ratio and debt-to-expenditure ratio. In overall terms, the model does a good job replicating the joint distribution, with a slight underestimation of the measure of households in the top quintile of leverage ratio and debt-to-expenditure ratio.

Regarding the policy functions, in the upper row of Figure 1.5 the solid lines correspond to the bond policy for the high (low) dividend shock in blue (red) and the average labor income shock as a function of the current asset holdings for three different values of the current bond holding $b^\#$. Additionally, the dashed lines represent the corresponding debt limits, and the black dashed lines correspond to the

Figure 1.4: Joint leverage ratio and debt-to-expenditure ratio distribution



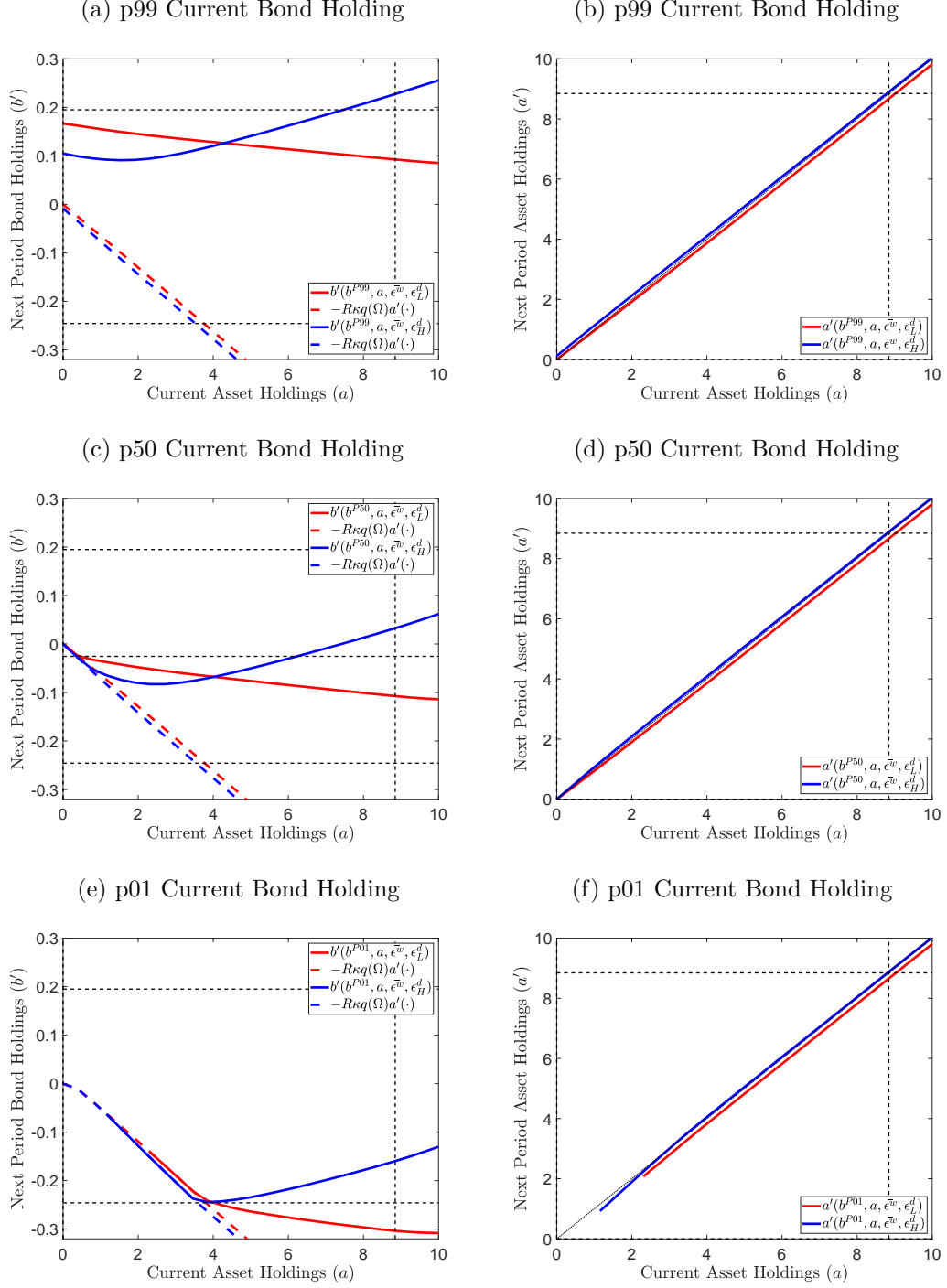
Notes: Solid lines correspond to the simulated distribution of the stationary model. Dashed lines correspond to the distribution for Mexican households in 2005.

bottom 1% and top 99% percentiles of bond and asset holdings obtained from the model's simulated time series. The figure, shows that for low dividend shocks (red lines) a household lowers their bond holdings (or gets more debt) as they increase their asset holdings. This effect is stronger for constrained households, as shown in panels c) and e). As described in Section 1.5.1, the *asset-wealth trade-off* generates the convex form of the bond policy for high dividend shocks (blue lines). For asset-poor households, as they increase their assets, they also lower their bond holdings (or get more debt if the holdings are negative) and there is a certain level for which the dividend risk exposure overcomes the benefit from more debt capacity that makes the households increase their bond holdings. Regarding the lower row of the figure, we can see the asset policy function that is highly linear and behaves as expected: for high-dividend shocks the households accumulate more assets, and for low-dividend shocks the households de-accumulate assets.

Moreover, in Figure 1.6 we show similar bond and asset policies but now as a function of the current bond holdings. In the left column, we can see the standard bond policies under a binding debt limit. Panel a) shows the policy for a high-asset holder. Here we can see that the debt limit is not binding for the states within the 1 and 99th percentiles. However, as we move to lower asset holdings, in Panel c) and e), we can see that the LtV becomes binding when households accumulate enough debt. With respect to the cross-sectional fire-sales in the model, in the right column we can see that households accumulate less assets as they increase their debt holdings. However, this relation is highly strengthened (households incur in fire-sales) when the debt limit becomes binding. This can be seen using panels c) and d) and also panels e) and f). There are strong declines in the asset holdings (panels d) and f)) in the states where the bond holdings reach the debt limit (panels c) and e)).

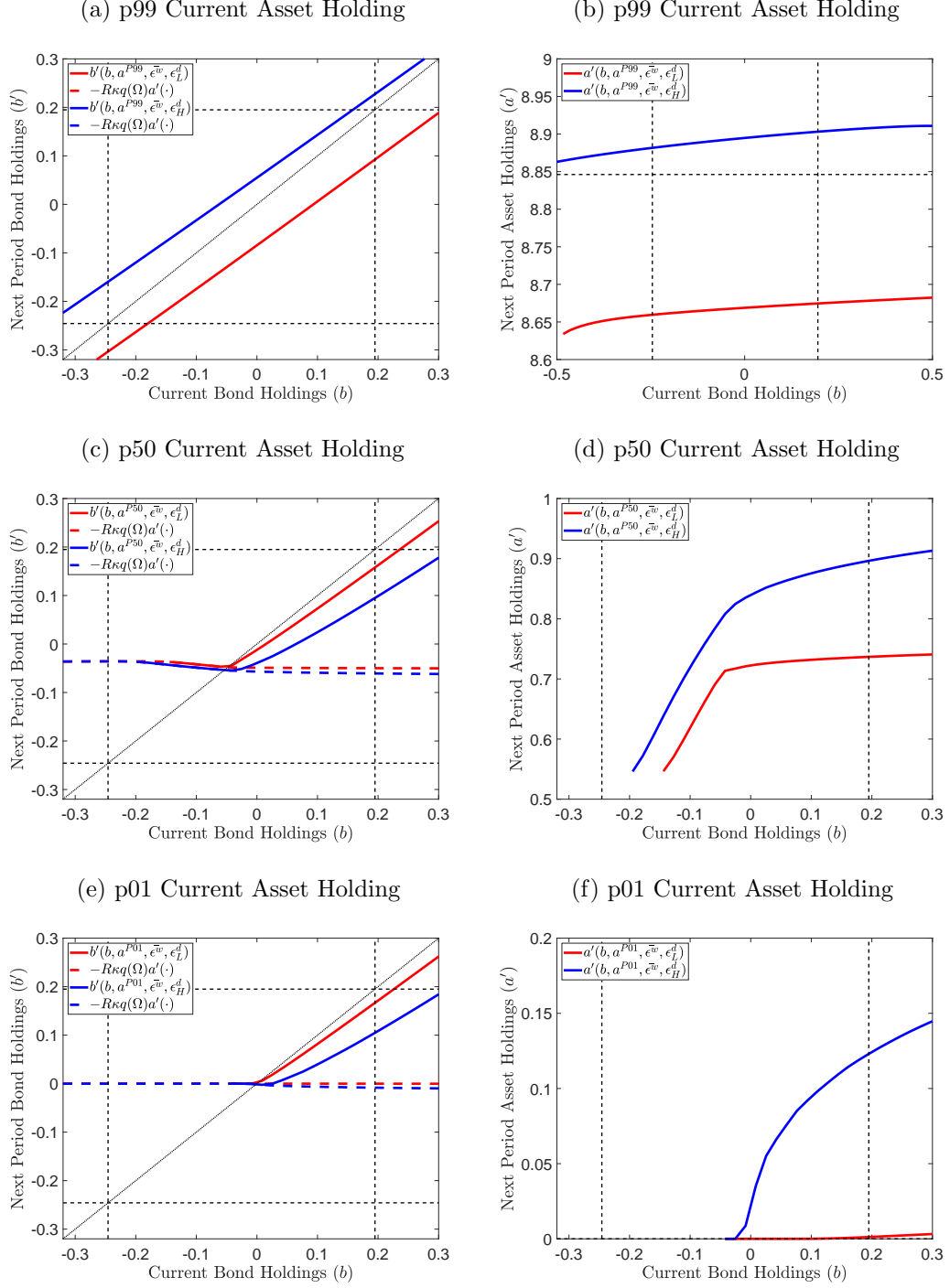
Additionally, in Figure 1.7 we show the difference between the bond policy func-

Figure 1.5: Stationary bond and asset policies as a function of current asset holdings



Notes: For a current bond holding $b^\#$ and mean labor shock $\bar{\epsilon}^w$, the left (right) column corresponds to the bond (asset) policies, the solid blue (red) line corresponds to the policy function with the high (low) dividend shock and the dashed blue (red) line corresponds to the debt limit with the high (low) dividend shock. Black dashed lines correspond to the bottom 1% and top 99% percentiles of bond and asset holdings obtained from the model's simulated time series. Black dotted lines correspond to the 45-degree line. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

Figure 1.6: Stationary bond and asset policies as a function of current bond holdings

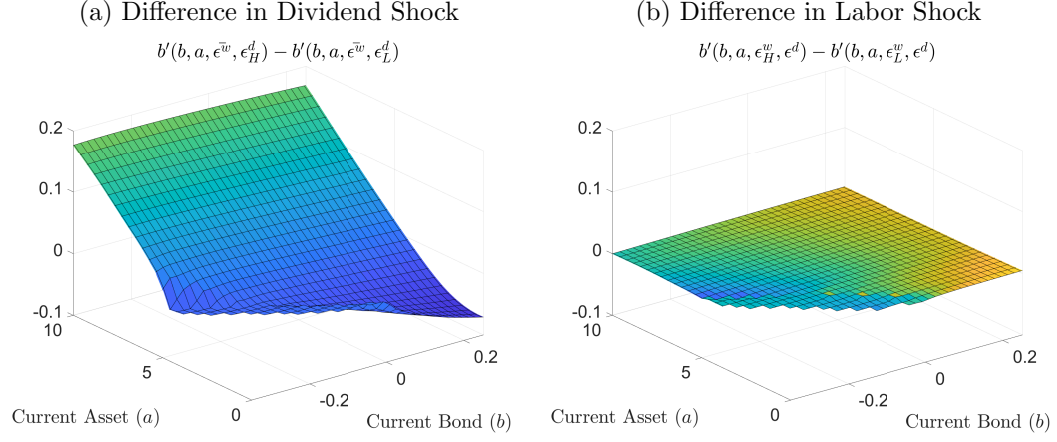


Notes: For a current bond holding $b^\#$ and mean labor shock $\bar{\epsilon}^w$, the left (right) column corresponds to the bond (asset) policies, the solid blue (red) line corresponds to the policy function with the high (low) dividend shock and the dashed blue (red) line corresponds to the debt limit with the high (low) dividend shock. Black dashed lines correspond to the bottom 1% and top 99% percentiles of bond and asset holdings obtained from the model's simulated time series. Black dotted lines correspond to the 45-degree line. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

tions and the dividend shocks in panel a) and labor income shocks in panel b). We can see a positive and increasing difference in the next period bond holdings between the high and low dividend productivities as we move to higher current asset holdings (Figure 1.7.a). This means that when the idiosyncratic dividend realization is high, the household optimally chooses also larger bond holding for the next period. Moreover, this difference is kept almost constant (only increases close to the debt limit) across the current bond holdings. In contrast, in Figure 1.7.b we can see that the difference in the bond policy function between the high and low idiosyncratic labor productivity realization is positive but close to zero and constant throughout all the feasible state space. Similarly, in Figure 1.8 we show the difference between the asset policy functions and the dividend shocks in panel a) and labor income shocks in panel b). We can see a positive and increasing difference in the next period asset holdings between the high and low dividend productivities as we move to higher current asset holdings (Figure 1.8.a). However, for high enough asset values, this positive difference becomes relatively constant. Moreover, this difference is kept almost constant (only increases close to the debt limit) across the current bond holdings. Finally, similarly to the bond policy function, in Figure 1.8.b we can see that the asset holding difference between the high and low idiosyncratic labor productivity realization is positive but close to zero and constant throughout all the feasible state-space.

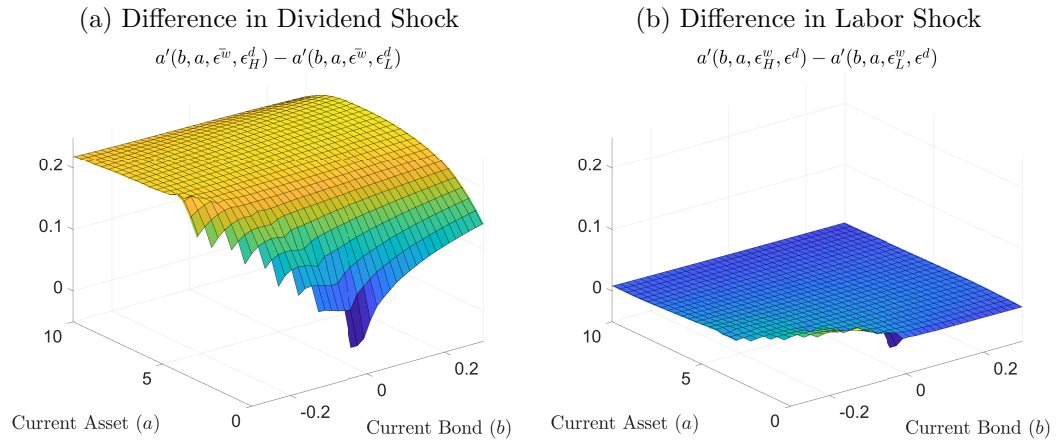
In summary, we used the stationary model to show the cross-sectional behavior of households. We can see that households with high-dividend shocks will accumulate more assets and, while they are still asset-poor, they de-accumulate bonds. Once they become asset-rich, because of the *asset-wealth trade-off*, they start accumulating more bonds (Figure 1.5). This behavior generates wealthy unconstrained households that drive the dampening cross-sectional effect. Moreover, we also show that households de-accumulate assets as they increase their debts, and that this relation strengthens

Figure 1.7: Effect of Non-insurable Individual Shocks in the Bond Policy



Notes: $\bar{\epsilon}^w$ and $\bar{\epsilon}^d$ correspond to the mean shock values. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

Figure 1.8: Effect of Non-insurable Individual Shocks in the Asset Policy



Notes: $\bar{\epsilon}^w$ and $\bar{\epsilon}^d$ correspond to the mean shock values. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

(households incur in fire-sales) when the debt limit is reached, driving the strength of the amplifying effect (Figure 1.6). Note that the representative-agent model would miss both effects. First, since there are no individual shocks, every household will behave in the same way. Hence, they either want to sell or want to buy more assets. Second, in that model, the average debt constraint multiplier will be the same as the individual debt multiplier, while in the heterogeneous-agents model, although fewer households could be constrained, they could have a stronger multiplier given the individual states. Finally, we used the stationary solution for simplicity and to avoid the extra aggregate states that would be needed in the aggregate risk model.

1.6.3 Aggregate Risk Model

To solve the aggregate risk model, we adapt the *non-trivial* market clearing algorithm proposed by Krusell and A. A. Smith (1997) to a small-open-economy framework. Specifically, we use the current aggregate net foreign asset position, $B \equiv \int_0^1 b^i di$, and the current interest rate $R - 1$ to forecast the next period's net foreign asset position B' and the domestic asset price q . This algorithm is computationally intensive since the market clearing asset price depends on the whole distribution of asset holdings and not only on the aggregate holdings (which are constant). For this reason, to obtain a simulated time series, each period, we use the aggregate law of motions to forecast the next period's aggregate net foreign asset position and the next period's asset's price. With these forecasts, we then solve a fixed-point problem for every period, which gives as solution the equilibrium market clearing price.¹⁸ The solution

¹⁸See Appendix 1.A.2 for a description of the solution algorithm.

of the aggregate law of motions are:

$$\begin{aligned} B' &= -0.005 + 0.870 B + 0.054 (R - 1), \quad R^2 = 0.99 \\ q &= 0.517 + 0.126 B - 0.301 (R - 1), \quad R^2 = 0.92 \end{aligned} \tag{1.12}$$

Simulation and Event Study of Sudden Stops

Using the solution to the aggregate law of motions, we simulate a panel of 1,000 households for 6,000 periods and drop the first 1,000 periods. Table 1.11 reports long-run moments of the main macro aggregates from the model with heterogeneous-agents and a representative-agent version without idiosyncratic risk and a lower leverage limit, κ that matches the same average leverage ratio of 0.11. Regarding the mean of the variables, the current account as a percentage of GDP is zero for both models. Average consumption is 8 percent higher, and the asset price is 40 percent higher in the heterogeneous-agents model. Since households do not need to self-insure against idiosyncratic shocks in the representative-agent model, there are less precautionary savings and less demand for the domestic asset. This equilibrium effect lowers the average asset price and tightens the aggregate financial conditions, lowering average consumption. Regarding the standard deviations, although the current account is 2.5 times more volatile and the asset price is 3.9 times more volatile in the representative-agent economy, consumption volatility is 17% larger in the heterogeneous-agents economy. This result comes from the larger consumption adjustments that high leveraged households have to do when they get hit by a negative shock. The heterogeneous-agents model shows high and positive first-order autocorrelations, which are in line with the data (see Mendoza, 2010). Lastly, regarding crisis episodes, we identify Sudden Stops as the periods in which the current account is 2 standard deviations above its historical mean, which is a common practice in empirical work (see Calvo,

Izquierdo, and Talvi, 2006). The last row of the table reports the probability of Sudden Stop events. In the heterogeneous-agents economy, a less volatile current account compared to the representative-agent economy, lowers the threshold to identify Sudden Stops and increases its frequency.

Table 1.11: Business Cycle Statistics

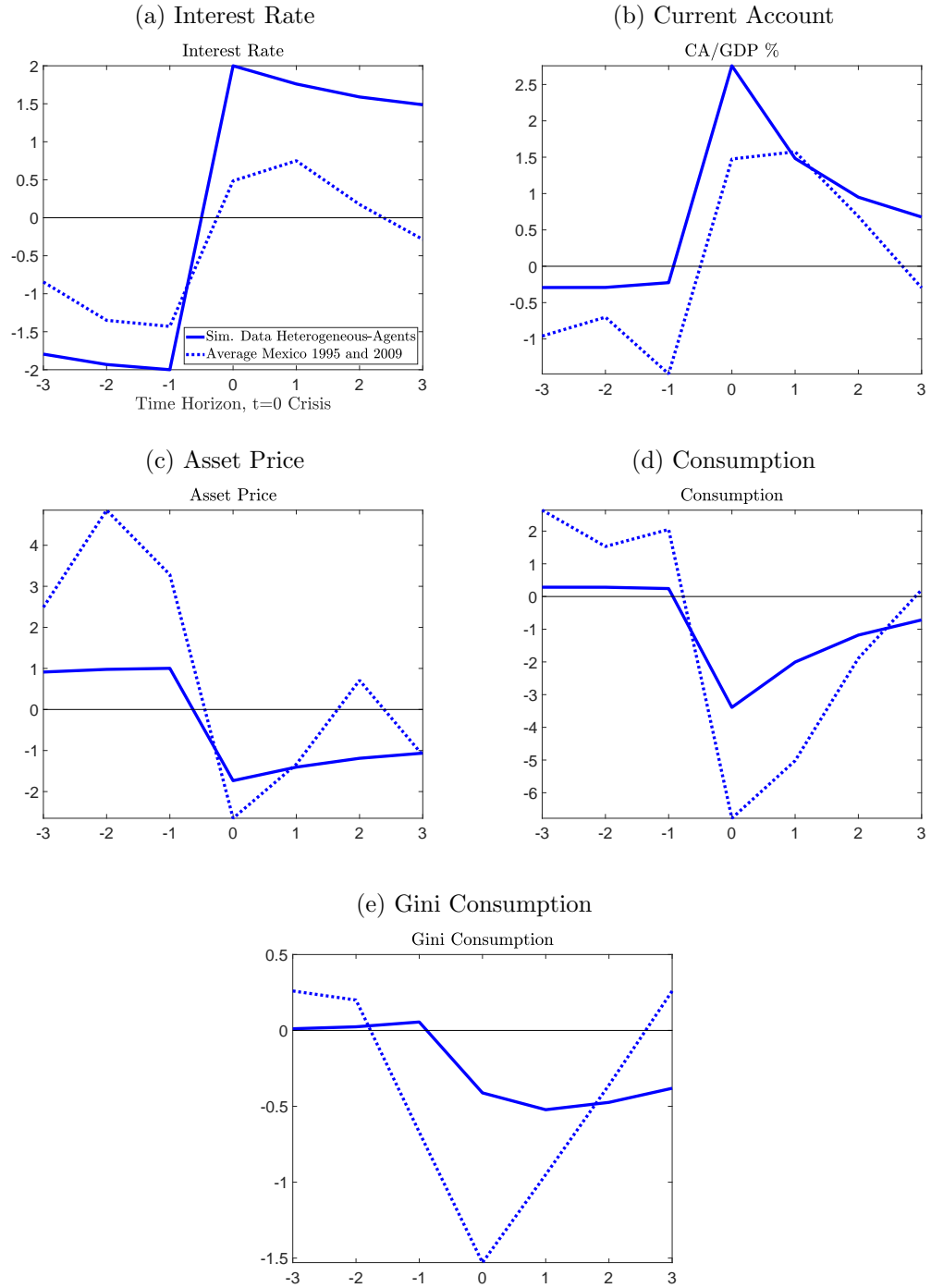
	Heterogeneous-agents	Representative-agent
Mean		
CA/GDP %	0.00	0.00
Consumption	0.12	0.11
Asset Price (q)	0.52	0.36
Standard deviation (in percent)		
CA/GDP %	0.89	2.01
Consumption	2.50	2.14
Asset Price (q)	1.23	4.24
First-order autocorrelation		
CA/GDP%	0.54	-0.99
Consumption	0.88	-0.77
Asset Price (q)	0.83	-0.60
Prob. of Sudden Stops	2.3%	1.4%

Notes: Sudden Stop episodes are defined as the periods where the current account as a percentage of GDP is 2 standard deviations above its mean.

To construct the event study of the simulated Sudden Stops, we average across all the identified crisis periods. Figure 1.9 shows the percent deviations from the steady state where the crisis period corresponds to $t = 0$. The average of the simulated crisis episodes in the heterogeneous-agents economy corresponds to the solid lines and the average of the data for Mexico around 1995 and 2009 Sudden Stops corresponds to the dashed line.

Figure 1.9.a shows that the Sudden Stops occur when there is an interest rate increase. This is expected since the interest rate is the only source of aggregate uncertainty in this economy. However, note that not all the interest rate increases cause

Figure 1.9: Event Study of a Sudden Stop



Notes: Solid lines correspond to the simulated data using the heterogeneous-agent model calibrated to Mexico, dotted lines correspond to the average of the Mexican data around the 1995 and 2009 Sudden Stops. Panels a), b) and e) correspond to the level difference to the long-run mean. Panels c) and d) correspond to percentage point deviations from the long-run average.

a crisis. Specifically, the long-run probability of a Sudden Stop in the simulated economy is 2.3%. In 1.9.b we can see that a crisis episode is preceded by periods with current account below the long-run average. Then, when the crisis happens ($t = 0$) there is a sharp reversal in the current account which means that international capital stops flowing into the economy. Consistent with the data, the crisis is persistent and takes more than 3 years for the international capital to flow back into the economy. Regarding the asset price drop, in 1.9.c we can see that the simulated price is 1.7% below the steady state which is below the asset price index for Mexico and in 1.9.d we can see that the model is able to generate a large and persistent aggregate consumption drop. Finally, 1.9.e shows that the model is able to capture a decline in consumption inequality during the crisis measured with the Gini coefficient, consistent with the data.

Regarding the differentiated individual effects during a Sudden Stop, in Tables 1.12 and 1.13 we show the dynamics of the asset holdings and consumption according to the leverage ratio and wealth of the households in a similar way as the results presented in Section 1.3.2. We can see that the model does a good job capturing the dampening effect coming from the wealthy unconstrained households that buy assets during a crisis and relieve the downward pressure on the price. In particular, these households increased by 3.8% their asset holdings during the crises. Moreover, in line with the empirical evidence on the amplifying effect, the financially constrained wealthy households are the ones that fire-sale the most their assets during the crisis and decreased their asset holdings by 9.8%. Although in the model, the households in decile IX of the leverage ratio do not sell their assets, we can see that they increase in a smaller amount than the low-leveraged households. Hence, the model is able to capture both cross-sectional effects. In Table 1.13, we see that, in line with the empirical evidence, households with larger leverage ratios decrease the most their

consumption. Hence, the model captures the heterogeneous consumption dynamics coming from the different leverage ratio levels and that crisis do not affect every household in the same way.

Table 1.12: Median % Asset Holdings Change in a Crisis

Leverage Ratio	Net Wealth	
	I-IX (Non-Wealthy)	X (Wealthy)
I-VIII (Low-LR)	-0.1	3.8
IX (High-LR)	1.7	2.3
X (Very High-LR)	0.8	-9.8

Notes: Ordered in the period of the crisis.

Table 1.13: Median % Consumption Change

Leverage Ratio	Pre-Crisis Period	Crisis Period
I	0.0	-1.5
II-IX	0.0	-2.0
X	0.4	-4.3

Notes: Ordered in the period previous to the crisis.

Lastly in Table 1.14, we show percent deviations from the steady state of the current account as a percentage of the GDP, consumption and the asset price for Mexico and different simulated economies. Columns (1) and (2) show the observed deviations in 1995 and 2009 for Mexico, respectively. In column (3) we show the heterogeneous-agents model calibrated to an emerging economy (Mexico). We can see that in the benchmark calibration, the asset price drop is smaller than the consumption drop, consistent with the data. Finally, in column (4) we show the representative-agent version of the model in which there is no idiosyncratic risk, and the leverage ratio limit, κ , is reduced to match the average leverage obtained in the heterogeneous-agent economy. Comparing columns (3) and (4) we can see that in the heterogeneous-agents economy the dampening effect dominates and asset prices drop less. However, there

is a larger adjustment in aggregate consumption mainly driven by the most leveraged households (see Table 1.13).

Table 1.14: Comparison of Dynamics during Sudden Stops

	(1)	(2)	(3)	(4)
	Mexico	Mexico	Het. Agents	Rep. Agent
	1995	2009	Benchmark EE	Match Mean Lev. Ratio
CA / GDP p.p.	2.6	0.4	2.8	0.3
Consumption	-8.3%	-5.3%	-3.4%	-1.3%
Asset Price (q)	-3.7%	-1.8%	-1.7%	-3.0%

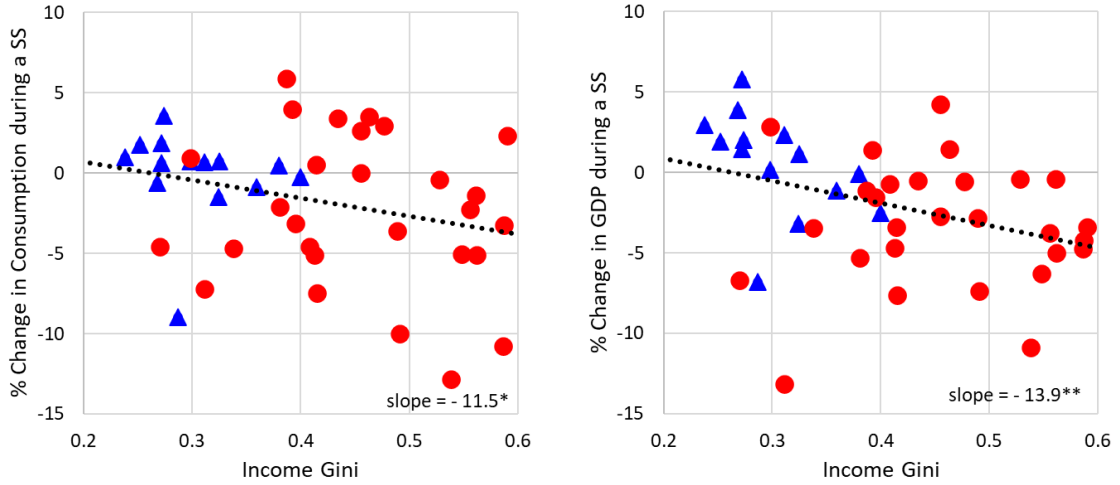
Notes: Sudden Stop episodes are defined as the periods where the current account as a percentage of GDP is 2 standard deviations above its mean.

Effect of a Lower Variance in the Dividend Risk

In this section, we compare the severity of Sudden Stops in economies with different degrees of inequality. Figure 1.10 shows descriptive evidence that crises are more severe in more unequal economies. The figure shows a scatter plot with the percentage change in consumption and in GDP during Sudden Stops for different economies (advanced in triangle and emerging in circle) against their income Gini index. This evidence suggests that emerging economies are more unequal and that there is a negative correlation between both variables.

To quantitatively assess the effects of lower income inequality, we calibrate the model to an advanced economy where the dividend risk is one-half of the benchmark emerging-markets model. In Figures 1.11 and 1.12 we show the event study analysis for the same history of individual and aggregate shocks for the two calibrations: the emerging economy from the previous section in solid lines and the advanced economy with the same calibration but with half variance in the dividend risk in dashed lines. The results during the crises, summarized in Table 1.15, show that in the version of the model calibrated to an advanced economy (dashed lines), the average net foreign

Figure 1.10: Severity of Sudden Stops and Inequality



Notes: Triangle (circle) markers correspond to advanced (emerging) economies. Dates of Sudden Stop episodes come from Bianchi and Mendoza (2020). Gini index measures income inequality, larger numbers mean larger inequality (income instead of wealth is used due to the availability in a larger sample of countries). *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: The World Bank.

debt position is twice as large, consumption drops 0.8 percentage points less and asset prices drop 0.4 percentage points less. Hence, the model predicts that in economies with less dividend return inequality, the economy supports larger debt positions and Sudden Stop crises are less severe, as observed in the data,

Table 1.15: Sudden Stop Deviations: Different Heterogeneous Economies

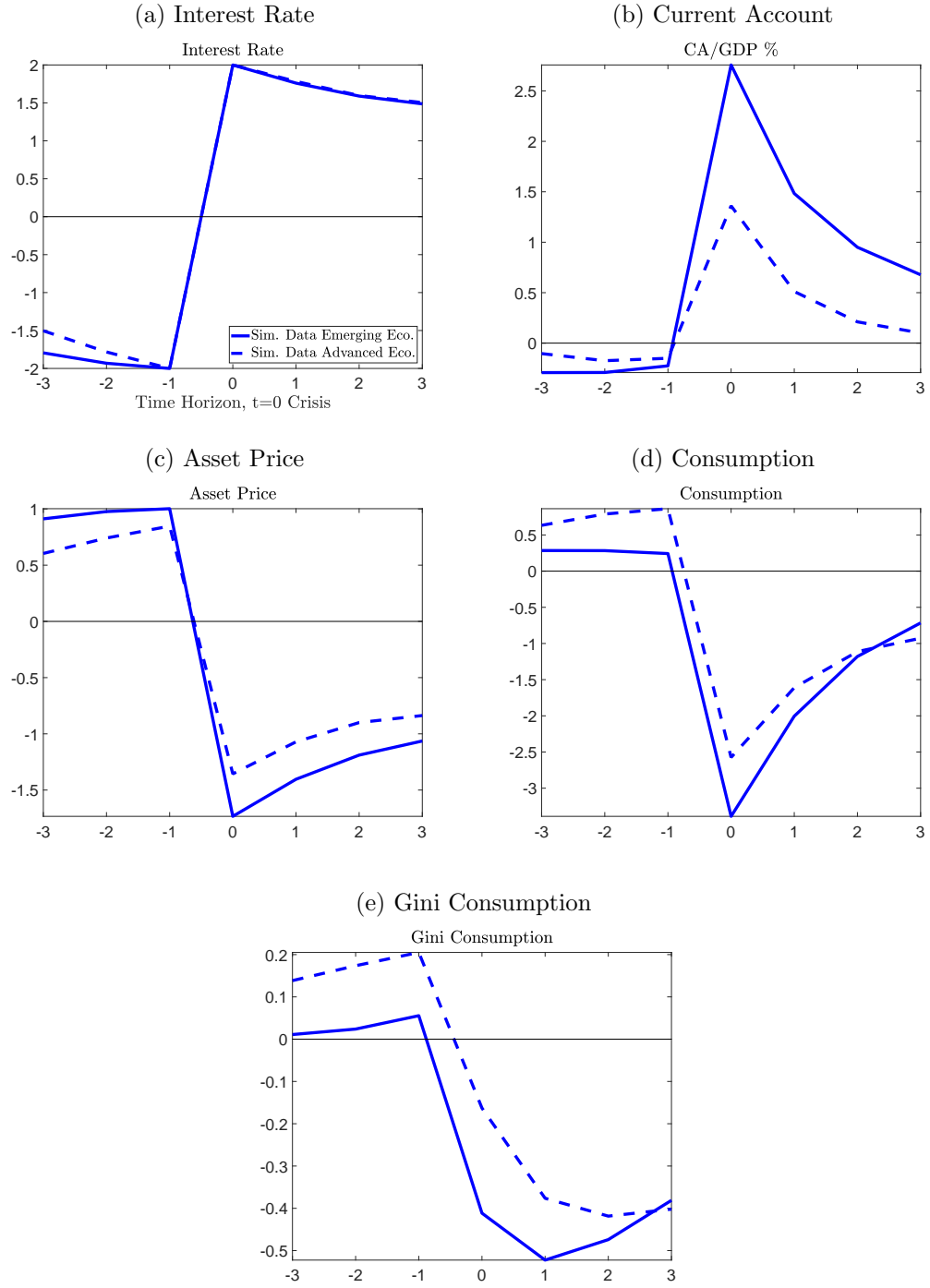
	(1)	(2)	(3)
	Het. Agents	Het. Agents	Het. Agents
	Benchmark EE	Adv Eco. ($\sigma^d/2$)	EE with div. tax
CA / GDP p.p.	2.8	1.4	1.9
Consumption	-3.4%	-2.6%	-2.4%
Asset Price (q)	-1.7%	-1.3%	-1.7%

Notes: Sudden Stop episodes are defined as the periods where the current account as a percentage of GDP is 2 standard deviations above its mean.

Effect of a Dividend Income Tax

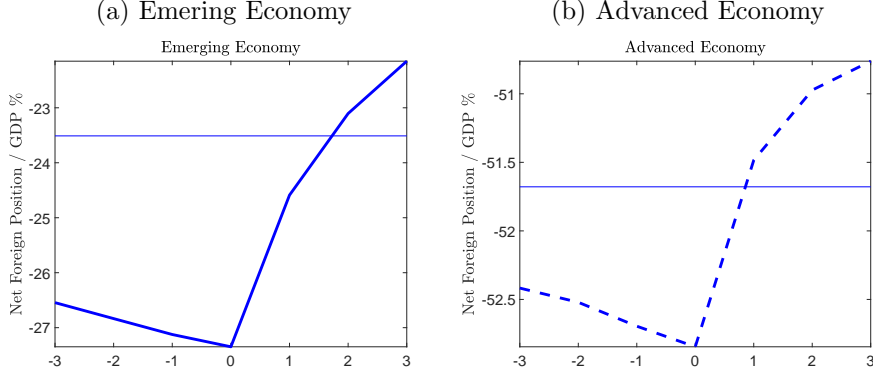
According to the OECD (2018), Mexico is one of the countries with the lowest tax rates. The marginal effective tax rate in Mexico for bank deposits and dividends is

Figure 1.11: Event Study of a Sudden Stop in Simulated Economies



Notes: Solid lines correspond to the simulated data using the heterogeneous-agent model calibrated to an emerging economy (Mexico) and dashed lines to the heterogeneous-agent model calibrated to an advanced economy which has one half the variance in the dividend risk. Panels a), b) and e) correspond to the level difference to the long-run mean. Panels c) and d) correspond to percentage point deviations from the long-run average.

Figure 1.12: Net Foreign Asset Position Event Study of a Sudden Stop in Simulated Economies



Notes: Solid blue horizontal lines correspond to the long-run averages.

around zero (negative for low-income households) while the OECD average rate is close to 30%. In this section, we use the proposed model to study the effect of a redistributive dividend income tax. Specifically, the government taxes the household's dividend returns at a constant (across periods and households) rate $\tau^d = 50\%$ and redistributes the tax revenue through lump-sum transfers T_t .¹⁹ The government follows a balanced budget every period which results in a time-varying transfer function $T_t = \int_0^1 a_t^i d_t^i \tau^d di$. The budget constraint of household i becomes

$$c_t^i + R_t^{-1} b_{t+1}^i + q_t(a_{t+1}^i + \Phi(a_{t+1}^i, a_t^i)) = w_t^i + a_t^i(q_t + d_t^i(1 - \tau^d)) + b_t^i + T_t. \quad (1.13)$$

The economy with a redistributive dividend income tax experiences more frequent but less severe crises. In particular, the probability of a Sudden Stop increases from 2.3% to 2.5% and the current account reversal is 0.9 percentage points smaller (see column (3) of Table 1.15). The intuition for this result is the following. In an economy with a positive redistributive dividend tax, households have a less potent

¹⁹Although the 50% rate is larger than the OECD average, we take it as a comparison benchmark to the previous section where we reduced the dividend variance by 50%.

precautionary savings motive since they are effectively less exposed to the dividend risk. Hence, they demand fewer bonds (or more debt if the bond holdings are negative) and less domestic assets. Given the lower aggregate demand for the domestic asset, its equilibrium price drops to clear the market. On average, the asset price is 54% smaller because of the dividend tax. Because the smaller asset price tightens the debt limit for every household (the pecuniary externality), the long-run share of financially constrained households increases from 3.4% to 8.2%. This effect increases the economy's exposure to movements in the international interest rate and generates more frequent crises.

Although the crises are more frequent, the domestic absorption change is less severe. In particular, aggregate consumption drops 1 percentage point less in the economy with the dividend tax. Because the financially vulnerable households have effectively less debt given the smaller asset price that tightened the debt limit, their bond adjustment is smaller, and together with the government transfers, the drop in consumption is less severe.

In Figure 1.13 we show graphically how the behavior of the households change given the introduction of the dividend tax in the stationary equilibrium. Specifically, the figure shows the economy's stationary bond policy functions with a dividend tax rate equal to 50% in red and with a rate equal to 0% in blue. The bond policies are represented for a current bond holding equal to the 1% most indebted household. We can see two effects coming from the *asset-wealth trade-off*. First, given that with a positive dividend tax there is a redistribution, the households effectively face a lower dividend risk. This effect, lowers the excess exposure channel for asset-rich households, lowering their next period bond holdings (increasing their debt positions). Hence, for high values of the current asset holding, the bond policy in the economy with the dividend tax (red) is below the bond policy in the economy without dividend

tax (blue). Second, since there is an aggregate decrease in the risk exposure, the precautionary savings motive for every household is less potent. Hence, the equilibrium asset price is lower due to the lower precautionary demand of the asset. This asset price effect tightens the debt constraint for every household. Hence, the financially vulnerable households (the constrained or close to becoming constrained households) effectively borrow less. Lastly, in Tables 1.16 and 1.17 we compute the dynamics of the asset holdings and consumption during crises. We can see how the fire-sale effect is stronger in the economy with a positive redistribution tax compared to Tables 1.12 and 1.13. However, given the redistribution, financially constrained households have to adjust by much less their consumption.

Figure 1.13: Stationary Bond Policies without and with a Dividend Income Tax

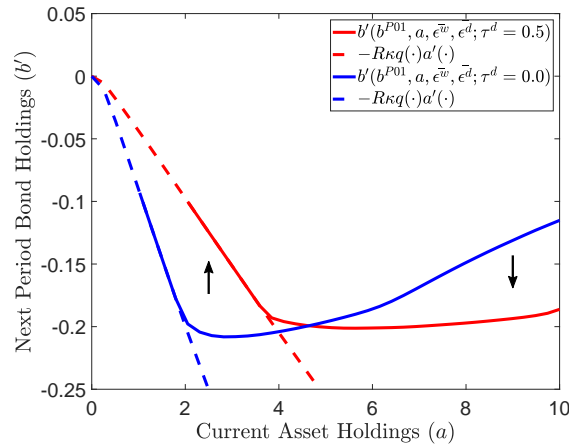


Table 1.16: Median % Asset Holdings Change in a Crisis with Dividend Tax

Leverage Ratio	Net Wealth	
	I-IX (Non-Wealthy)	X (Wealthy)
I-VIII (Low-LR)	-0.3	3.1
IX (High-LR)	1.8	-0.8
X (Very High-LR)	1.2	-7.6

Notes: Ordered in the period of the crisis.

Table 1.17: Median % Consumption Change with Dividend Tax

Leverage Ratio	Pre-Crisis Period	Crisis Period
I	0.0	-1.3
II-IX	0.0	-1.8
X	0.3	-1.0

Notes: Ordered in the period previous to the crisis.

1.7 Conclusions

This paper studies the cross-sectional dimension of the debt-deflation mechanism that triggers endogenous financial crises of the Sudden Stop type. This dimension is relevant for the macroeconomy for two reasons. First, there is a dampening effect on the deflation of asset prices coming from the unconstrained wealthy households who buy depressed assets, relieving the downward pressure on asset prices. Second, there is an amplifying effect on the asset price deflation coming from the financially vulnerable households who fire-sale assets, generating a stronger downward pressure on asset prices. Because these two cross-sectional effects move asset prices in opposite directions, the cross-section and inequality role during crises is quantitatively ambiguous. Hence, this paper examines how the frequency and severity of Sudden Stops crises are affected by inequality in an economy.

Using panel data for Mexican households, we document micro-data evidence that supports both effects. Specifically, the 2009 crisis had different effects on the households depending on the composition of their balance sheets. The real estate holdings of low-leveraged wealthy households increased 59.4% during the crisis while wealthy households with high-leverage fire-sold and decreased the most their assets during the crisis. Additionally, in terms of the consumption dynamics, high-leverage households decreased their expenditures 6.2% while non-leveraged households increased 5.4% during the crisis. These heterogeneous asset and consumption dynamics during the

crisis highlight the importance of the opposing forces that are missed when the financial crises are studied under a representative-agent framework. For this reason, we proposed a model to quantify a Sudden Stop's effect on asset prices and consumption, accounting for the household's heterogeneity in their balance sheet.

Using the proposed asset-pricing Bewley model of a small-open-economy, we find that in a version of the model calibrated to an emerging economy (Mexico), the model can explain Sudden Stops' key stylized facts and generate persistent current account crises. Regarding the cross-sectional forces, the dampening effect dominates and asset prices drop less during Sudden Stop episodes in heterogeneous-agents economies. In contrast to the representative-agent framework, the model produces an empirically plausible leverage ratio distribution and generates persistent current account reversals with larger drops in consumption driven by the most leveraged households. Moreover, calibrating the model to an advanced economy where the dividend risk is one-half of the benchmark emerging-markets model, the average net foreign debt position is twice as large, consumption drops 0.8 percentage points less, and asset prices drop 0.4 percentage points less. Hence, the model predicts that in economies with less dividend return inequality, larger debt positions are supported, and Sudden Stop crises are less severe, as observed in the data. Additionally, an economy with a redistributive dividend income tax experiences more frequent but less severe crises. This result comes from an equilibrium effect that lowers the asset price, tightening the financial conditions. Hence, increasing the share of constrained households that effectively hold less debt and adjust their consumption less.

References

- Aiyagari, S Rao and Mark Gertler (1991). “Asset returns with transactions costs and uninsured individual risk”. In: *Journal of Monetary Economics* 27.3, pp. 311–331.
- (1999). “Overreaction of asset prices in general equilibrium”. In: *Review of Economic Dynamics* 2.1, pp. 3–35.
- Angeletos, George-Marios (2007). “Uninsured idiosyncratic investment risk and aggregate saving”. In: *Review of Economic dynamics* 10.1, pp. 1–30.
- Benhabib, Jess, Alberto Bisin, and Shenghao Zhu (2011). “The distribution of wealth and fiscal policy in economies with finitely lived agents”. In: *Econometrica* 79.1, pp. 123–157.
- (2015). “The wealth distribution in Bewley economies with capital income risk”. In: *Journal of Economic Theory* 159, pp. 489–515.
- Bianchi, Javier (2016). “Efficient bailouts?” In: *American Economic Review* 106.12, pp. 3607–59.
- Bianchi, Javier and Enrique G Mendoza (2018). “Optimal time-consistent macroprudential policy”. In: *Journal of Political Economy* 126.2, pp. 588–634.
- (2020). “A Fisherian approach to financial crises: Lessons from the Sudden Stops literature”. In: *Review of Economic Dynamics*.
- Bordo, Michael D and Christopher M Meissner (2012). “Does inequality lead to a financial crisis?” In: *Journal of International Money and Finance* 31.8, pp. 2147–2161.
- Bosch, Mariano and Julen Esteban-Pretel (2015). “The labor market effects of introducing unemployment benefits in an economy with high informality”. In: *European Economic Review* 75, pp. 1–17.

- Calvo, Guillermo A, Alejandro Izquierdo, and Ernesto Talvi (2006). “Sudden stops and phoenix miracles in emerging markets”. In: *American Economic Review* 96.2, pp. 405–410.
- Damodaran, Aswath (2013). “Equity risk premiums: determinants, estimation and implications”. In: *Stern School of Business. New York University*.
- Fagereng, Andreas et al. (2020). “Heterogeneity and persistence in returns to wealth”. In: *Econometrica* 88.1, pp. 115–170.
- Fisher, Irving (1933). “The debt-deflation theory of great depressions”. In: *Econometrica: Journal of the Econometric Society*, pp. 337–357.
- Gomes, Diego BP, Felipe S Iachan, and Cezar Santos (2020). “Labor earnings dynamics in a developing economy with a large informal sector”. In: *Journal of Economic Dynamics and Control* 113, p. 103854.
- Guerrieri, Veronica and Guido Lorenzoni (2017). “Credit crises, precautionary savings, and the liquidity trap”. In: *The Quarterly Journal of Economics* 132.3, pp. 1427–1467.
- Guntin, Rafael, Pablo Ottonello, and Diego J Perez (2020). “The Micro-Anatomy of Macro-Consumption Adjustments”. In:
- Güvenen, Fatih (2007). “Learning your earning: Are labor income shocks really very persistent?” In: *American Economic Review* 97.3, pp. 687–712.
- (2009). “An empirical investigation of labor income processes”. In: *Review of Economic Dynamics* 12.1, pp. 58–79.
- Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante (2010). “The macroeconomic implications of rising wage inequality in the United States”. In: *Journal of Political Economy* 118.4, pp. 681–722.

- Heaton, John and Deborah J Lucas (1996). “Evaluating the effects of incomplete markets on risk sharing and asset pricing”. In: *Journal of political Economy* 104.3, pp. 443–487.
- Hubmer, Joachim, Per Krusell, and Anthony A Smith Jr (2020). “Sources of US wealth inequality: Past, present, and future”. In: *NBER Macroeconomics Annual 2020, volume 35*. University of Chicago Press.
- Huo, Zhen and José-Víctor Ríos-Rull (2016). “Financial frictions, asset prices, and the great recession”. In: *CEPR Discussion Paper No. DP11544*.
- Jappelli, Tullio and Luigi Pistaferri (2017). *The economics of consumption: theory and evidence*. Oxford University Press.
- Kaplan, Greg and Giovanni L Violante (2014). “A model of the consumption response to fiscal stimulus payments”. In: *Econometrica* 82.4, pp. 1199–1239.
- Krueger, Dirk, Kurt Mitman, and Fabrizio Perri (2016). “Macroeconomics and household heterogeneity”. In: *Handbook of Macroeconomics*. Vol. 2. Elsevier, pp. 843–921.
- Krusell, Per and Anthony A Smith (1997). “Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns”. In: *Macroeconomic dynamics* 1.2, pp. 387–422.
- Kumhof, Michael, Romain Rancière, and Pablo Winant (2015). “Inequality, leverage, and crises”. In: *American Economic Review* 105.3, pp. 1217–45.
- Leyva, Gustavo and Carlos Urrutia (2020). “Informality, labor regulation, and the business cycle”. In: *Journal of International Economics*, p. 103340.
- Maliar, Lilia, Serguei Maliar, and Fernando Valli (2010). “Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm”. In: *Journal of Economic Dynamics and Control* 34.1, pp. 42–49.

- Meghir, Costas and Luigi Pistaferri (2004). “Income variance dynamics and heterogeneity”. In: *Econometrica* 72.1, pp. 1–32.
- Mehra, Rajnish and Edward C Prescott (1985). “The equity premium: A puzzle”. In: *Journal of monetary Economics* 15.2, pp. 145–161.
- Mendoza, Enrique G (2010). “Sudden stops, financial crises, and leverage”. In: *The American Economic Review* 100.5, pp. 1941–1966.
- Mendoza, Enrique G, Vincenzo Quadrini, and Jose-Victor Rios-Rull (2009). “Financial integration, financial development, and global imbalances”. In: *Journal of Political economy* 117.3, pp. 371–416.
- Mendoza, Enrique G and Katherine A Smith (2006). “Quantitative implications of a debt-deflation theory of sudden stops and asset prices”. In: *Journal of International Economics* 70.1, pp. 82–114.
- Mendoza, Enrique G and Sergio Villalvazo (2020). “FiPIt: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints”. In: *Review of Economic Dynamics* 37, pp. 81–102.
- Morelli, Salvatore and Anthony B Atkinson (2015). “Inequality and crises revisited”. In: *Economia Politica* 32.1, pp. 31–51.
- OECD (2018). *Taxation of Household Savings*, p. 240. DOI: <https://doi.org/https://doi.org/10.1787/9789264289536-en>. URL: <https://www.oecd-ilibrary.org/content/publication/9789264289536-en>.
- Rubalcava, Luis and Graciela Teruel (2006). “Mexican family life survey”. In: *Second Round*. www.ennvih-mxfls.org.
- (2013). “Mexican family life survey”. In: *Third Round*. www.ennvih-mxfls.org.
- Storesletten, Kjetil, Christopher I Telmer, and Amir Yaron (2004). “Consumption and risk sharing over the life cycle”. In: *Journal of monetary Economics* 51.3, pp. 609–633.

Storesletten, Kjetil, Christopher I Telmer, and Amir Yaron (2007). “Asset pricing with idiosyncratic risk and overlapping generations”. In: *Review of Economic Dynamics* 10.4, pp. 519–548.

Townsend, Robert M (1995). “Consumption insurance: An evaluation of risk-bearing systems in low-income economies”. In: *Journal of Economic perspectives* 9.3, pp. 83–102.

1.A Appendix

1.A.1 The 2009 Mexican Sudden Stop at the Aggregate Level

A Sudden Stop is a fast and large outflow of international capital (Calvo, Izquierdo, and Talvi, 2006). Hence these types of episodes are characterized by large Current Account (CA) movements.²⁰ In this Appendix, we use aggregate data to show the Sudden Stop that the Mexican economy experienced in 2009.

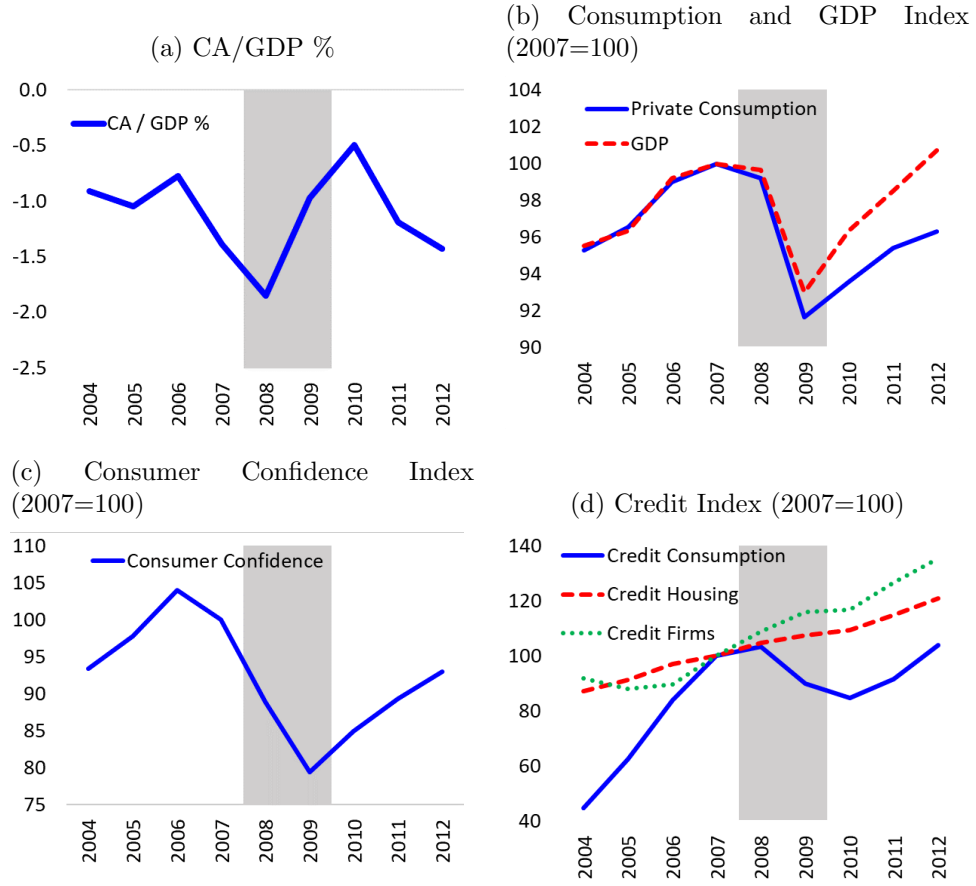
In Figure 1.14 we can see that the current account deficit reversed around 1.5 percentage points of GDP. Also, GDP and consumption declined, there was a drop in the consumer confidence and a decline in consumption credit while firm and housing credit was not affected.

On the prices side, in Figure 1.15 we see that there was a large decline in the stock market, house prices decelerated and remained constant for about 4 years since the crisis burst, the J.P. Morgan EMBI+ spread that measures the Mexican sovereign bonds risk increased about 2 percentage points and there has a large depreciation of the Mexican peso against the dollar.

The aggregate dynamics shown in this Appendix are not particular to Mexico.

²⁰Some Sudden Stop episodes have even registered CA reversals. Meaning that the economy transits from having a negative CA (foreign capital entering the economy) to positive CA surpluses (capital leaving the economy).

Figure 1.14: Quantities and Consumption determinants



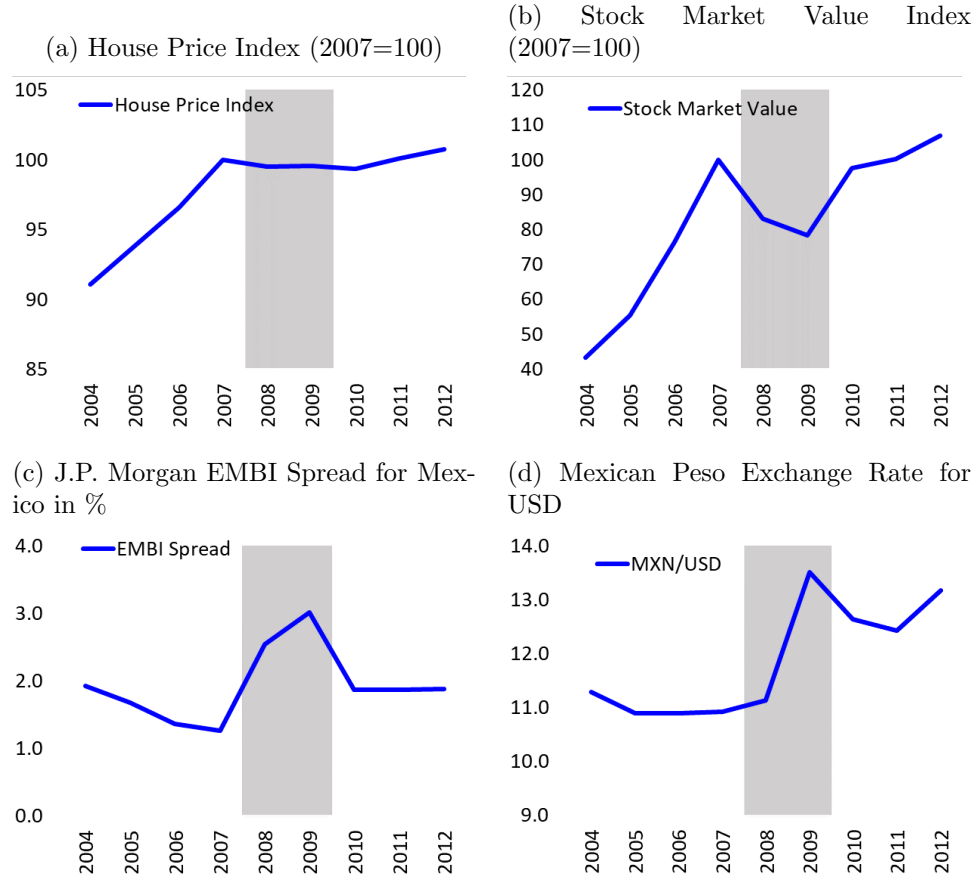
Notes: The grey area corresponds to the crisis. Source: INEGI, World Bank, Banxico.

See Bianchi and Mendoza (2020) for a recent survey of Sudden Stop episodes both among advanced and emerging economies.

1.A.2 Solution Algorithm

In this Appendix we describe the solution method. Building from Krusell and A. A. Smith (1997), we adapt their *non-trivial* market clearing algorithm to a small-open-economy framework. In particular, instead of solving problem 1.5, we solve:

Figure 1.15: Asset Prices



Notes: The grey area corresponds to the crisis. Source: Sociedad Hipotecaria Federal, Moodys Analytics, INEGI, World Bank.

$$\begin{aligned}
 \tilde{v}(b, a, \epsilon^w, \epsilon^d, B, \epsilon^R, q) &= \max_{\{c, b', a' \geq 0\}} u(c) + \beta \mathbb{E}[v(b', a', \epsilon^{w'}, \epsilon^{d'}, B', \epsilon^{R'})] \quad s.t. \\
 c + R(\epsilon^R)^{-1}b' + q(a' + \Phi(a', a)) &= \epsilon^w w + a(q + \epsilon^d d) + b, \\
 R(\epsilon^R)^{-1}b' &\geq -\kappa q a', \\
 \Phi(a', a) &= \frac{\phi}{2}(a' - a)^2 \\
 q' &= \gamma_q^0 + \gamma_q^1 B + \gamma_q^2 (R - 1) \\
 B' &= \gamma_B^0 + \gamma_B^1 B + \gamma_B^2 (R - 1)
 \end{aligned} \tag{1.14}$$

Where we replaced the full household distribution Ω with the aggregate bond position $B = \int b \, d\Omega$, and market clearing in the asset holdings is achieved using a fixed-point iteration on q such that $\bar{K} = \int a'(\cdot) \, d\Omega$. Then, the solution algorithm follows the simulation method described in Krusell and A. A. Smith (1997).

Chapter 2

FDI Flows and Sudden Stops in Small Open Economies

BY SERGIO VILLALVAZO MARTÍN[†]

2.1 Motivation

Most of the Sudden Stops (SS) literature has focused on emerging economies neglecting that from 1990 to 2016 there have been 16 SS episodes in advanced economies.¹ Although, for the past almost three decades advanced economies have been experiencing episodes of capital outflows that have been associated only to emerging and fragile economies, the probability of experiencing a SS in an advanced economy is 20 percent smaller than in an emerging economy.² Is there any difference other than income levels driving these probabilities? This paper contributes to closing the literature

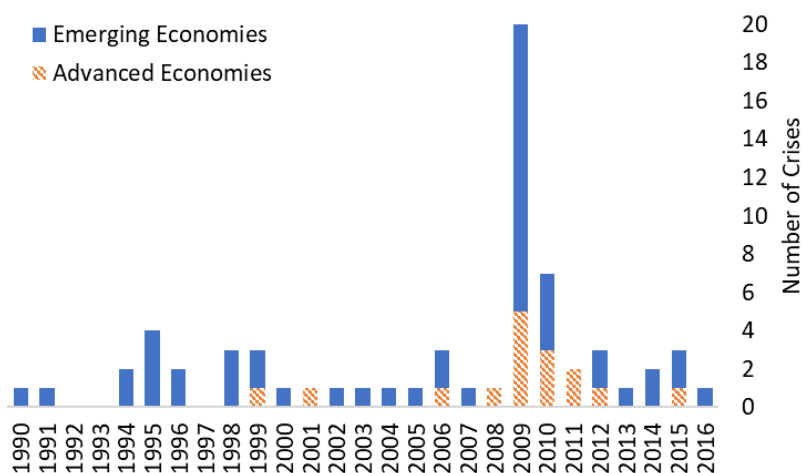
[†]University of Pennsylvania.

¹See Figure 2.1 and J. Bianchi and Mendoza (2020) for a recent survey. The terms emerging and upper-middle income will be used interchangeably, as well as the terms advanced and high income. The income threshold is taken from the World Bank classification.

²Specifically, using the panel database constructed in this paper, the probability in an advanced economy is 2.3 percent while for an emerging economy is 2.9 percent.

gap by studying and contrasting SS episodes in advanced and emerging economies, focusing on the role of Foreign Direct Investment (FDI) through the lens of a small open economy framework.

Figure 2.1: Number of Sudden Stops by year and by classification of economies.



This paper explores the complementarities between FDI and Portfolio Investment (PI). The mechanism through which both accounts interact is the following. As FDI enters an economy, the borrowing capacity of the economy increases because the amount of available collateral increases through two channels. First, the direct effect in emerging economies is that a fraction of the foreign stock of capital is subject to expropriation risk and thus can be used as collateral to increase the borrowing capacity of the economy, and second, the indirect effect is that FDI flows affect the domestic price of capital and thus change the market value of all the available collateral in the economy (both domestic and foreign capital stocks).³ Both channels move the borrowing capacity of the economy in the same direction: less (more) foreign capital tightness (loosens) the borrowing constraint. This spillover effect from FDI to the borrowing constraint amplifies the negative shocks that hit an economy that is close

³Discussion and evidence of the fact that expropriation risk is only present in emerging economies will be presented in Sections 2.2 and 2.4.

to its debt limit. The above mechanism, together with a price deflation mechanism similar to the one introduced by Mendoza (2010), will generate endogenous Sudden Stop crises.

A Sudden Stop is defined as a large, fast, and unlikely outflow of capital in the Financial Account (FA) of the Balance of Payments Identity (similar definitions have been used by Calvo, Izquierdo, and Talvi, 2006, Mendoza, 2010, among others). At the aggregate FA level, every country that experiences a SS is similar since they all register a large capital outflow. However, after decomposing the FA into its main components there are significant differences between emerging and advanced economies (see Figures 2.2 and 2.3 for a small sub-sample of economies from both groups). Advanced economies have net flows of FDI as a percentage of GDP that fluctuate around zero (some years positive and some years negative) while emerging economies tend to have only negative net flows (inflows of capital). This paper will focus on this difference between advanced and emerging economies and will explore the effects of FDI movements during crises.

A sizable literature, starting more than 25 years ago with Backus, Kehoe, and Kydland (1992) and Baxter and Crucini (1995), has documented how international financial markets are a transmission mechanism of business cycles among economies. A strand of this literature, closely related to this paper, has studied business cycles in small open economies (see Heathcote and Perri, 2002 and Garcia-Cicco, Pancrazi, and Uribe, 2010). However, the main focus of our paper is considerably narrower. We will measure the effect of the different characteristics of international capital flows, between emerging and advanced economies, on the dynamics and probability of a balance of payments crisis. In particular, this paper will study the differences between FDI and PI flows. Regarding the former, Albuquerque, Loayza, and Servén (2005) study how an increase in FDI is related to global factors and higher integration

in capital markets. In that paper, the authors argue that FDI may look similar to equity flows, although, the former does not depend on the existence of developed stock markets. For this reason, it seems more appropriate to use FDI given that capital liberalization has occurred in different stages of development for each country. They find that global factors have become more relevant and that these factors can explain better the dynamics of FDI since some local factor risks can be hedged due to the increase in financial liberalization. In line with the authors findings about the importance of global factors, our analysis will includes the international interest rate level and volatility as exogenous global factors. However, regarding local factors, this paper documents the importance of the expropriation risk for FDI and its effect during crises.

The two main components of the FA are Portfolio Investment and Direct Investment (FDI), which differ in maturity and volatility. As noted by Albuquerque (2003), FDI is a less volatile long-term position given natural constraints to rapidly withdraw illiquid investments. On the other hand, Portfolio Investments have shorter maturity, since technological advances provide additional flexibility for the investments to leave an economy faster. Hence, from the perspective of international investors, the current opportunity cost of an investment (i.e. the international interest rate) is not the only moment affecting investment decisions, but also the current state of international volatility and its effect on future returns. C. M. Reinhart and V. R. Reinhart (2001) document that when volatility in the US interest rate is high, net FDI flows to emerging economies are 23 percent smaller. Therefore, introducing an element of time-varying volatility in the international risk provides a deeper understanding of the dynamics behind the different capital flows and the effect of having different FDI flows.

In terms of structural modeling, some characteristics of the FDI on which this

paper focuses on have been previously documented in the literature. In Albuquerque (2003), the author argues that FDI is less volatile than other financial flows and that non-FDI flows are shorter-term investments facing less physical constraints to movement, and thus making it easier to flee a jurisdiction. The author proposes a model with enforcement constraints in which FDI is partly inalienable to the extent that it comprises intangible assets, and portfolio flows are subject to expropriation due to the lack international enforcement mechanisms. The author finds that more financially constraint economies should borrow more relatively through FDI. The model in our paper differs from his since we model portfolio flows to be subject to a loan-to-value constraint and we study the mechanism through which the risk of expropriation of FDI in emerging economies affects the debt capacity of the economy. Hence, in our paper, the risk of expropriation is one of the key elements that explain the difference between advanced and emerging economies. According to the World Bank (2017), 5 percent of foreign investment is expropriated in emerging economies and this risk is a major concern for multinationals when they choose where, when, and how much to invest. The World Bank, through the Global Investment Competitiveness group, surveyed executives of multinational corporations with investments in developing countries. They find that over 90 percent of all investors say that legal protections are critically important in the decision process of investing abroad. These guarantees include laws that protect against expropriation, breaches of contracts and arbitrary government conducts.

Regarding local factors, this paper contributes to the literature on emerging economies expropriation risk that has been studied by Thomas and Worrall (1994), Antras, Desai, and Foley (2009), Hajzler (2012), among others, by analyzing the effects of the risk of FDI expropriation on Sudden Stop crises. In particular, we study the complementarities between FDI and Portfolio flows, the relations between FDI

and the debt capacity of the domestic economy, and the different exposure to crises between advanced and emerging economies. Lastly, our paper quantifies the effect of this risk in a small open economy model with financial frictions in which Sudden Stops arise endogenously.

Another closely related strand of the literature focuses on the real effect of time-varying volatility of the international interest rate. Justiniano and Primiceri (2008) estimate a large-scale DSGE model that allows time variation in the volatility of the structural innovations and conclude that volatility has decreased dramatically in the postwar era having a large effect on investment. Following this line of research, Fernández-Villaverde et al. (2011) document how changes in the volatility of the interest rate can have an effect on output, consumption, investment and hours worked even when the interest rate level does not change. The present paper contributes to this growing literature by introducing time-varying volatility to a small open economy model with an endogenous occasionally-binding constraint and quantifies the effect of time-varying volatility on the dynamics of the Balance of Payment accounts and GDP during a Sudden Stop.

All the previous works have studied real business cycles long-run moments. However, the focus of this paper is on the dynamics of Sudden Stops. Hence, our model will build on Mendoza (2010) work which introduces the debt-deflation mechanism to study SS episodes (Uribe and Schmitt-Grohé, 2017 provide a textbook treatment of open economy models with collateral constraints). We follow this set-up to analyze the FDI channel during Sudden Stop crises. In particular, this paper studies the different characteristics of the capital flows between advanced and emerging economies and its effect on the dynamics of the economies during crises.

The rest of the paper is structured as follows. Section 2.2 describes the panel database constructed and shows empirical evidence on the importance of the FDI

channel. In Section 2.3, we propose a small open economy model with financial frictions that incorporates both types of international capital flows: Portfolio Investment and Direct Investment subject to expropriation risk, and allows for time-varying volatility in the international interest rate. Then, Section 2.4 presents the quantitative results from running simulations with calibrated parameters for each type of economy. We quantify how much of the differences in the probability of a Sudden Stop observed in the data can be accounted by the FDI channel and also perform an impulse response exercise to quantify the effects of temporary and permanent increases in the volatility of the interest rate and the expropriation risk. Finally, Section 2.5 concludes.

2.2 Empirical Evidence

The first point this paper aims to make is that Sudden Stop crises happen also in advanced economies. To accomplish this, we construct a panel database of 31 advanced and 75 emerging economies from 1990 to 2016. The economies were selected according to the classification of the World Bank of high income economies (advanced) and upper-middle income economies (emerging).⁴ Following Calvo, Izquierdo, and Talvi (2006), we identify a SS episode as a large outflow of capital from an economy. Specifically, a change in the Financial Account as a percentage of GDP 2 standard deviations above the historical mean in a year will be considered a SS episode. Figure 2.1 shows the number of crises per year for both groups of economies. There have been 16 crises in advanced and 50 crises in emerging. This evidence suggests that SS are not a phenomenon exclusive of emerging economies although they are more probable than in advanced economies. Moreover, the distribution of capital outflows

⁴See the Appendix for the list of countries in each group.

in emerging economies shows fatter tails. The average Kurtosis coefficient for an emerging economy is 2.0 while for advanced economies is 0.9. This evidence suggests that there is a fundamental difference between both groups of economies regarding how net international flows enter and leave the economies. Figure 2.1 also highlights the importance of global factors since SS crises do not happen in isolation; there seems to be a clustering of episodes during the mid 90's, early 2000's, and during the great recession years. Given this evidence, we state the following Fact 1.

Fact 1: The probability of a SS in advanced economies is 20 percent smaller than in emerging and the distribution of outflows in emerging economies shows fatter tails.

2.2.1 Differences in the FDI flows and the capital stock

Although at the aggregate level of the Financial Account a crisis seems similar between economies (see Figure 2.4.b), a decomposition of the FA suggests fundamental differences between both groups of economies. The mean net FDI to GDP flow for emerging economies is -3.9 percent (negative sign corresponds to inflows) while for advanced economies is -0.3 percent, and the mean inflow FDI to GDP flow for both emerging and advanced economies is -5.1 percent.⁵ These percentages suggest that net FDI and inflow FDI are similar in emerging economies while very different in advanced. Moreover, the net FDI account in the former is mainly an inflow account: capital is only flowing into the economy. While for advanced economies, similar magnitudes of inflows and outflows of capital are registered such that the net FDI is around zero and even positive in some years. Hence, emerging economies only have inflows of capital while advanced economies attract capital and invest abroad

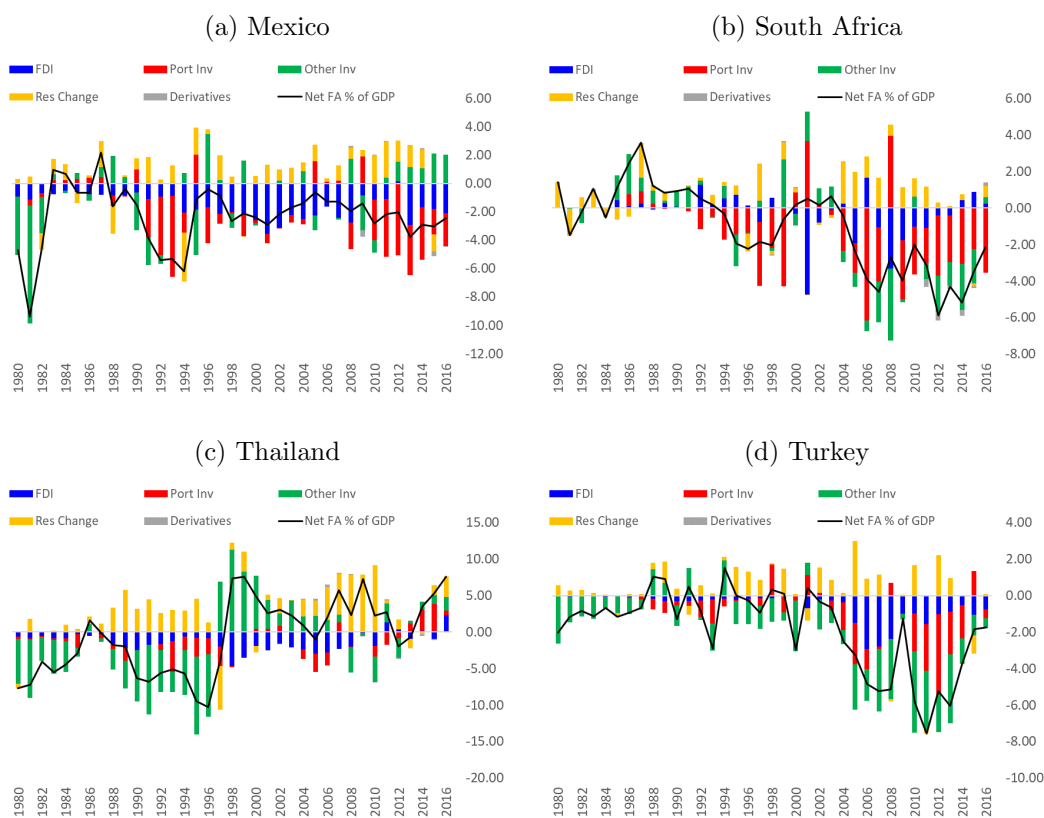
⁵To obtain the moments we averaged each country across time and then took the mean across countries.

approximately in the same magnitudes possibly due to diversification motives.

Figures 2.2 and 2.3 show the decomposition of the Financial Account for a subsample of 4 economies for each group. Emerging economies (Figure 2.2) consistently have negative FDI flows. This means that capital from abroad is flowing into the economy. As a global resource constraint would imply, this capital is coming from another economy, which most likely is an advanced economy. Figure 2.3 gives evidence that advanced economies have both positive and negative large flows of FDI. Hence, let Fact 2 be:

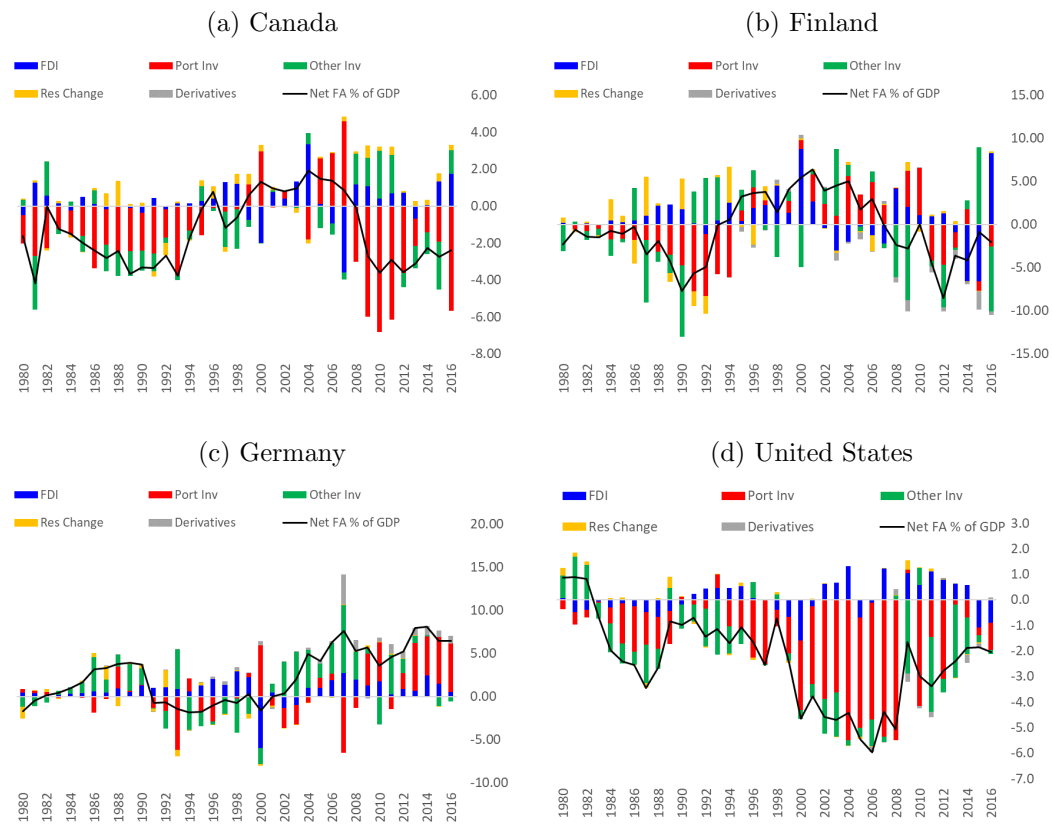
Fact 2: The mean net FDI as a percentage of GDP flow for emerging economies is -3.9 percent and for advanced is -0.3 percent.

Figure 2.2: Financial Account in Emerging Economies



Notes: Financial Account in a sample of emerging economies. Source: World Bank WDI and IMF.

Figure 2.3: Financial Account in Advanced Economies



Notes: Financial Account in a sample of advanced economies. Source: World Bank WDI and IMF.

Lastly, estimates of the total stock of capital in each group of economies also show significant differences. Advanced economies have a stock of capital to GDP ratio 15 percent larger than emerging economies.⁶ Given this evidence, we state Fact 3.

Fact 3: The mean capital to GDP ratio in advanced economies is 2.4 and in emerging economies is 2.1.

Facts 2 and 3 can be rationalized as follows: under a national aggregate production function with diminishing marginal returns to capital and no domestic investment, emerging economies that have smaller stocks of domestic capital relative to advanced economies will have a greater rate of return on capital and will attract a larger amount of international capital inflows.

These differences can be seen not only at a business cycle level among the whole sample but also during Sudden Stop episodes. Figure 2.4 shows median GDP, FA, FDI, and Portfolio plus Other Investments during crisis episodes for both classifications of economies. The graphs are centered around period 0 that corresponds to the period identified as a SS. Even when the method to identify a crisis does not include directly a drop in the GDP, Figure 2.4.a shows a drop in the cycle component of the GDP for both groups. In this sense, SS's are accompanied by declines in the production that are 1.5 percentage points more severe in emerging economies. We can see in Figure 2.4.b that at the aggregate level, the FA as a percentage of GDP follows a similar movement in both economies although, before the SS, emerging economies have a more negative position, of around 4 percentage points more than advanced economies. However, after decomposing into FDI and PI (that also includes Other Investments) we can see a clear difference between groups. On the Portfolio side (Figure 2.4.c), although both groups show similar movements, before

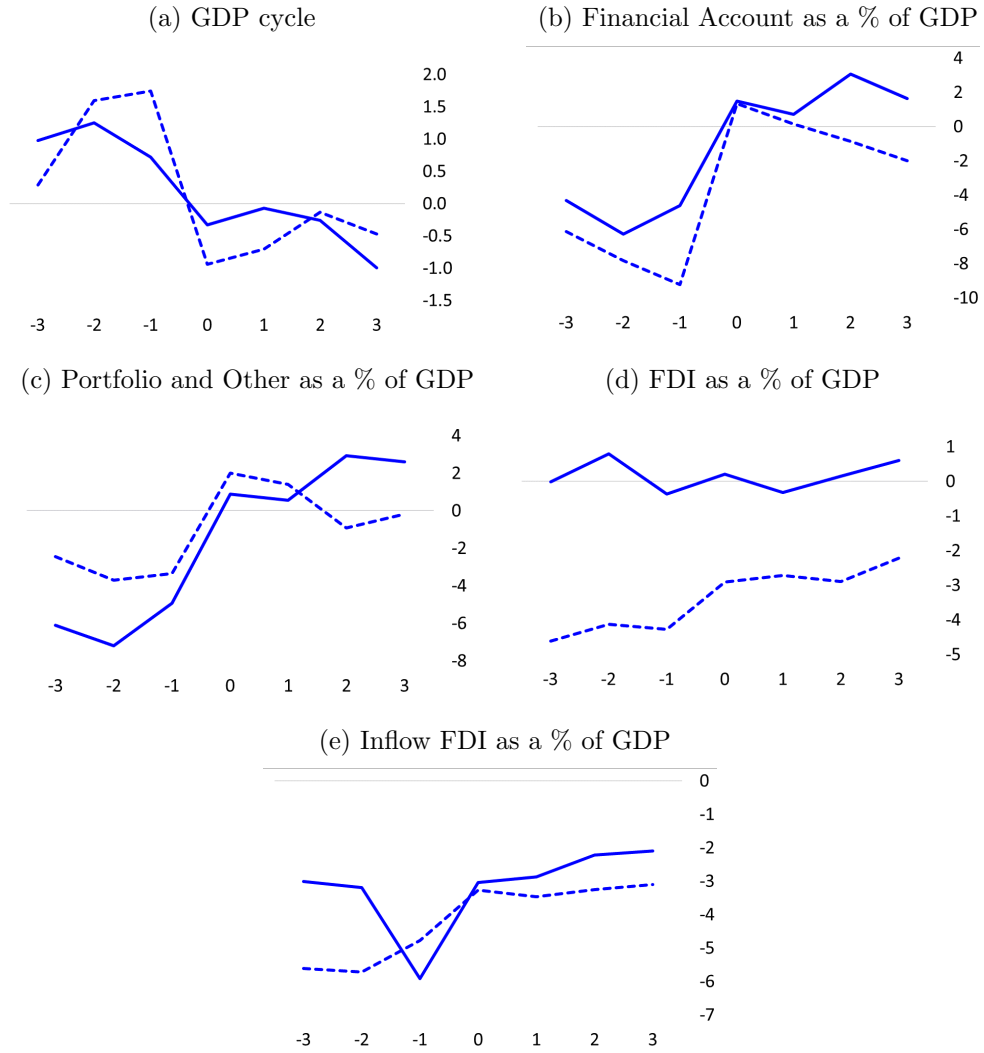
⁶Capital stock estimated are obtained from the IMF Investment and Capital Stock Dataset, see International Monetary Fund (2015).

the SS, advanced economies have a more negative position and the contraction during the crisis is larger. Figure 2.4.d shows two clear differences between both groups of economies: the FDI flows previous to a SS account for almost half of the FA deficit in emerging economies (4 percent) while for advanced economies the flows are close to zero, and emerging economies suffer a large correction in FDI the year of the SS (1.5 percentage points) while advanced economies can smooth it out.

This second difference might suggest that multinational corporations seem to behave different if they have invested in emerging or in advanced economies. Whenever there is a crisis in an emerging economy, international investors will move their FDI investments out of such economy while if the crisis happens in an advanced economy they are more resilient to move their investments. However, Figure 2.4.e suggests that this is not the case.

Figure 2.4.e shows the inflow FDI event study analysis for both groups of economies. The graph suggests that multinational corporations react in the same way in both groups of economies. Whenever the crisis hits the domestic economy, FDI investments are pulled out of the economy (independently if it is advanced or emerging). Hence, the difference between groups comes from domestic investors and relies on the fact that advanced economies have outflow FDI investments of the same magnitude as the inflows they receive and these outflows react and move in opposite ways to the inflows such that the net FDI account is around zero, even when the crisis hits the advanced economy. In this sense, outflow FDI investments serve as buffer savings in advanced economies that let them smooth their Financial Account account whenever the economy enters a Sudden Stop episode and possible prevents them from experiencing more severe crises more frequently.

Figure 2.4: Event Study of a Sudden Stop



Notes: Solid (dashed) lines correspond to advanced (emerging) economy.

2.2.2 Importance of the international volatility

The Financial Account records transactions that involve financial assets and liabilities that take place between residents and non-residents. Its two main components, FDI and Portfolio Investment, are different in nature. According to the International Monetary Fund (2013):

“Direct investment is a category of cross-border investment associated with a resident in one economy having control or a significant degree of influence on the management of an enterprise that is resident in another economy.”

and,

“Portfolio investment is defined as cross-border transactions and positions involving debt or equity securities, other than those included in direct investment or reserve assets.”

Hence, these accounts involve international transactions of different things. Portfolio investments are the exchanges of financial securities while Direct investments are the exchanges of control (ownership) of enterprises. From the perspective of an international investor (noted earlier by Albuquerque, 2003), FDI is a less volatile longer-term investment while Portfolio could be short-term. Given the different possible maturities of each investment, not only the current interest rate is relevant but also its volatility. Moreover, C. M. Reinhart and V. R. Reinhart (2001) find that when volatility in the US interest rate is high net FDI flows to emerging economies are 23 percent smaller.

Following the literature on high frequency data we construct a proxy of the volatility of the US interest rate using its realized volatility. Using average monthly series (intra-period information) we estimate the standard deviation for a year (period

length of analysis) and use it as a proxy for international volatility. Figure 2.5 shows the 3-Month Treasury Bill real rate for the US and its realized volatility.⁷

Having documented the importance of the FDI flows and the state of the international volatility to study Sudden Stop episodes, the next section will describe the proposed model that incorporates both elements.

2.3 Model

2.3.1 Environment

This paper proposes a standard real business cycle of a small open economy model (RBC-SOE) with an endogenous occasionally-binding constraint, a fixed domestic stock of capital, and foreign investment subject to expropriation risk. The model builds from Mendoza (2010) with two new elements: an FDI channel and time-varying volatility in the international interest rate.

The economy is inhabited by an infinitely lived household with preferences defined over stochastic sequences of consumption and labor $\{c_t, L_t\}$ for $t = 0, \dots, \infty$. The preference specification is:

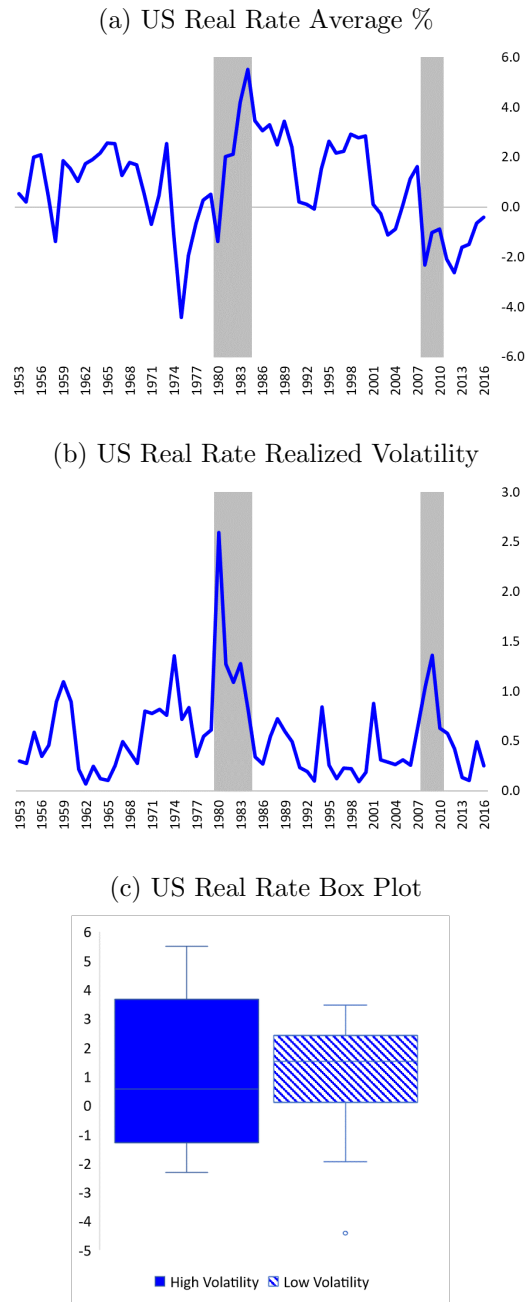
$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \right], \text{ where } u(c_t, L_t) = \frac{(c_t - \frac{L_t^\omega}{\omega})^{1-\nu}}{1-\nu}. \quad (2.1)$$

The GHH type utility function proposed by Greenwood, Hercowitz, and Huffman (1988) is commonly used in RBC-SOE models since the wealth effects on the labor supply are eliminated and a closed form expression for the labor supply can be obtained.

The representative household has access to a non-state-contingent bond, b_{t+1} , that

⁷The nominal rate was converted to a real rate using the past 12 months inflation.

Figure 2.5: US real interest rate and realized volatility



Notes: The gray area corresponds to high volatility periods. Source: FRED.

pays one unit in the next period with price equal to the inverse international interest rate factor, $q_t = (1 + r_t)^{-1}$. The household will choose sequences of consumption, supply of labor and bond positions to maximize her lifetime expected utility subject to the following period budget constraint:

$$c_t + q_t b_{t+1} = w_t L_t + r_{k,t} \bar{k} + b_t + T_t. \quad (2.2)$$

The agent income comes from the labor income, $w_t L_t$, plus the capital income from the fixed domestic stock of capital owned by the agent, $r_{k,t} \bar{k}$, plus any bond position coming from the previous period, b_t , plus any transfers from the government, T_t . On the expenditure side, the agent will buy consumption (numeraire good with normalized price equal to 1) goods, c_t , plus the next period bond position, b_{t+1} , multiplied by its price, q_t . However, next period bond position is subject to a collateral constraint:

$$q_t b_{t+1} \geq -\kappa q_{k,t} \bar{k} - \kappa_{f,t} q_{k,t} k_{f,t+1}. \quad (2.3)$$

The household will not be able to issue more debt (negative bond positions) than a constant fraction κ of the market value (the capital, both locally and foreign owned, has price $q_{k,t}$) of the fixed domestic capital stock, \bar{k} , plus a stochastic fraction $\kappa_{f,t}$ of the market value of the next period foreign stock of capital in the economy, $k_{f,t+1}$.⁸ The market value is the price of the capital multiplied by the corresponding stock of capital (i.e. for the domestic capital, the market value is $q_{k,t} \bar{k}$). The fraction $\kappa_{f,t}$ corresponds to the exogenous probability that the government expropriates the foreign capital.

The consumption good is produced by a single firm with a constant-returns-to-

⁸Following Mendoza (2010) and Mendoza and Villalvazo (2020), in the competitive equilibrium the price of capital will be obtained from Tobin's Q investment optimality condition: $q_{k,t} = \partial \tilde{I}_t / \partial K_{t+1}$.

scale production function, that uses labor and capital as production inputs, and is exposed to a stochastic total factor productivity (TFP) shock, $y_t = \exp(\epsilon_t)AK_t^\alpha L_t^{1-\alpha}$. Total capital demanded by the firm, K_t , is composed of the exogenously fixed domestic stock, \bar{k} , and an endogenous foreign stock (FDI), $k_{f,t}$, which are additive perfect substitutes: $K_t = \bar{k} + k_{f,t}$. The firm, which is owned by the household and has zero profits, chooses every period how much capital to rent at the competitive rate, $r_{k,t}$, and how much labor to demand for a competitive wage, w_t . Both input prices are taken as given by the firm. The TFP shock, ϵ_t , follows a first-order Markov process. The international interest rate, r_t , follows a stochastic process with time-varying volatility, σ_t , that follows a regime-switching process. The stochastic process's will be specified at the end of this section.

There is also an international investor that chooses sequences of foreign capital to invest in the economy and rent to the domestic firm (note that the rental rate will be such that the foreign capital market will clear), $k_{f,t}$ for $t = 1, \dots, \infty$, as to maximize the expected present discounted value of profits paid to their global shareholders (a similar setup was introduced in Mendoza and Smith, 2006) with the addition that the international investor takes into account the expropriation risk. The objective function of this investor is:

$$\sum_{t=0}^{\infty} \mathbb{E}_0 [M_t (r_{k,t} k_{f,t} (1 - \kappa_{f,t}) - (k_{f,t+1} - (1 - \delta)(1 - \kappa_{f,t}) k_{f,t}) + \Phi(k_{f,t+1}, k_{f,t}))], \quad (2.4)$$

given $k_{f,0}$ and where M_t is the stochastic discount factor used by the financial institution (we will assume $M_t = q_t = \frac{1}{1+r_t}$). The function $\Phi(k_{f,t+1}, k_{f,t}) = \frac{\phi}{2} \frac{(k_{f,t+1} - k_{f,t})^2}{k_{f,t}}$ corresponds to a standard quadratic adjustment cost function incurred by the international investor to move capital globally.

Lastly, the government will play a simple but crucial role of expropriating foreign

capital and transferring these resources to the agent in a lump-sum transfer T_t every period.

As noted above, the exogenous stochastic shocks of the model are four: the TFP shock ϵ_t , the international interest rate r_t , the international interest rate volatility σ_t , and the expropriation risk $\kappa_{f,t}$. The TFP shock will follow a standard independent AR1 process. The interest rate will follow an AR1 process with time-varying volatility:

$$r_t = (1 - \rho_{\sigma_r})\bar{r} + \rho_{\sigma_r}r_{t-1} + \sigma_t\epsilon_{r,t}, \quad \epsilon_r \sim N(0, 1).$$

The volatility, σ_t , will follow a regime-switching process between low and high periods of volatility. Finally, the probability of expropriation will also follow a regime-switching process between low and high probability of expropriation periods (independent of all the other processes).

2.3.2 Recursive competitive equilibrium

The individual state variables are today's bond position b , the foreign owned capital stock in the economy k_f , and the exogenous state vector of shocks composed by TFP shock, the international interest rate and its volatility and the probability of expropriation: $s = (\epsilon, r, \sigma, \kappa_f)$, and the aggregate state variable is today's aggregate total capital K . In the recursive formulation variables with a prime, $'$, correspond to the next period.

Household's problem:

$$v(b, s; K) = \max_{c, L, b'} u(c, L) + \beta \mathbb{E}[v(b', s'; K') | \sigma(s)] \quad s.t.$$

$$c + q(s)b' = w(s; K)L + r_k(s; K)\bar{k} + b + T(s; K) \text{ , Budget Constraint,}$$

$$q(s)b' \geq -\kappa q_k(s; K)\bar{k} - \kappa_f(s)q_k(s; K)k'_f(s; K) \text{ , Debt Constraint,}$$

$$K' = H_K(s; K) \text{ , Rational Expectations of the household.}$$

Let $\lambda(b, s; K) \geq 0$ be the multiplier on the budget constraint and $\mu(b, s; K) \geq 0$ on the debt constraint, then first order conditions are:

$$\begin{aligned} \left(c - \frac{L^\omega}{\omega}\right)^{-\nu} &= \lambda(b, s; K) \\ \left(c - \frac{L^\omega}{\omega}\right)^{-\nu} (-L^{\omega-1}) &= \lambda(b, s; K)w(s; K) \\ \beta \mathbb{E}[v_{b'}(b', s'; K') | \sigma(s)] &= \lambda(b, s; K)q(s) - \mu(b, s; K)q(s) \\ 0 &= \mu(b, s; K)(q(s)b' + \kappa(q_k(s; K)\bar{k}) + \kappa_f(s)(q_k(s; K)k'_f(s; K))). \end{aligned}$$

We can see from the last first order condition how the introduction of expropriation risk loosens the constraint on the maximum amount of debt that the economy can hold.

Firm's problem:

$$\max_{K, L} \exp(\epsilon(s))AK^\alpha L^{1-\alpha} - w(s; K)L - r_k(s; K)K$$

$$\Rightarrow \text{F.O.C.:}$$

$$r_k(s; K) = \alpha \exp(\epsilon(s))AK^{\alpha-1}L^{1-\alpha}$$

$$w(s; K) = (1 - \alpha) \exp(\epsilon(s))AK^\alpha L^{-\alpha}.$$

Foreign Investor's problem:

$$v_f(k_f, s; K) = \max_{k'_f > 0} r_k(s; K)k_f(1 - \kappa_f(s)) - I + \frac{1}{1 + r(s)} \mathbb{E}[v_f(k'_f, s'; K') | \sigma(s)] \quad s.t.$$

$$I = k'_f - (1 - \delta)k_f(1 - \kappa_f(s)) + \Phi(k'_f, k_f)$$

$$K' = H_K(s; K)$$

$$\Rightarrow \text{F.O.C.:}$$

$$1 + \Phi_1(\cdot) = \frac{1}{1 + r(s)} \mathbb{E}[r_k(s'; K')(1 - \kappa_f(s')) + (1 - \delta)(1 - \kappa_f(s')) + \Phi_2(\cdot) | \sigma(s)].$$

Where $\Phi(k'_f, k_f) = \frac{\phi}{2} \frac{(k'_f - k_f)^2}{k_f}$ and $\Phi_n(\cdot)$ corresponds to the first derivative of the adjustment cost function with respect to the n argument.

From the first order condition we can see how the introduction of the expropriation risk distorts the optimal decision of the international investor. In the current period, the investor takes into account that if there is a positive probability of being in a state with positive κ_f in the future, the expected return on the investments will be lower. Hence, optimality is achieved with a lower level of foreign capital (less FDI enters the economy).

Finally, the Recursive Competitive Equilibrium is given by the allocation functions $\{c(b, s; K), L(b, s; K), b'(b, s; K), k'_f(k_f, s; K), T(s; K)\}$, the price functions $\{w(s; K), r_k(s; K), q_k(s; K), q(s)\}$ and the functions $\{v(b, s; K), v_f(k_f, s; K), H_K(s; K)\}$ such that:

1. Given the prices, the functions $\{c(b, s; K), L(b, s; K), b'(b, s; K)\}$ solve the household's problem.
2. Given the prices, the firm maximizes profits.
3. Given the prices, the function $k'_f(k_f, s; K)$ solves the Foreign Investor's problem.

4. The price of the bonds satisfies $q(s) = (1 + r(s))^{-1}$ and the price of the capital satisfies Tobin's Q optimality condition $q_k(s; K) = \partial I(K', K) / \partial K'$

5. The capital market clearing condition is satisfied:

$$K = \bar{k} + k_f$$

6. The representative agent's condition is satisfied:

$$K' = H_K(s; K) = \bar{k} + k'_f(K - \bar{k}, s; K)$$

7. The government's budget is balanced:

$$T(s; K) = \kappa_f(s) k_f[r_k(s, K) + 1 - \delta].$$

2.4 Quantitative Analysis

In this section we report the results obtained after solving the model calibrated to an emerging and an advanced economy.

2.4.1 Calibration

The parameters of the utility function and the capital depreciation rate were taken from the literature with studies that used data from the Mexican economy. In particular, the risk aversion coefficient, ν , equal to 2 and the labor parameter that determines the wage elasticity of labor supply, ω , equal to 1.85 were taken from Mendoza (2010). The annual depreciation rate, δ , equal to 8.8 percent and was taken from García-Verdú (2005).

Regarding the parameters that were calibrated to match specific moments of the data, the discount factor, β , equal to 0.956 was calibrated to match the probability

of a Sudden Stop of 2.9 percent in emerging economies.⁹ The fix domestic capital stock, \bar{k} , for an emerging (advanced) economy was set to 1.90 (3.14) to match the average FDI to GDP percentage of -3.9 (-0.3) percent. The share of capital, α , was set to 0.23 to match the average capital to GDP ratio for an emerging economy of 2.1. The debt fraction of domestic collateral, κ , was set to 0.22 to match Mexico's Debt to GDP ratio of -35 percent. Lastly, the adjustment cost coefficient, ϕ , equals 8.5 to match the median ratio of Portfolio flows standard deviation to FDI flows standard deviation of 1.85 in emerging economies.

With respect to the exogenous process, the 3-Month Treasury Bill for the US was used as a proxy for the international interest rate and was converted to a real rate using the past 12 months inflation. Intra-period data (monthly) was used to construct period (yearly) realized volatility. The volatility process is assumed to follow a two-state regime-switching process. To identify the different volatility periods we divided the sample from 1953 to 1984 and from 1985 to 2016, the latter period is also known as the Great Moderation era. Then, high volatility periods were identified to start the first year in which the volatility was 2 standard deviations above the historical mean, for each sub-sample, and lasted all the subsequent years for which the volatility was a quarter of a standard deviation above the mean. The resulting high volatility episodes are from 1980 to 1984 which is known as the period of highly active monetary policies made by Federal Reserve Chairman Paul Volcker to control inflation and from 2008 to 2011 which were the years of the Great Recession.¹⁰ The low volatility episodes are from 1953 to 1979 and from 1985 to 2007 (see Figure 2.5). Finally, the volatility process calibration is set to capture the average duration of low volatility periods of 25

⁹The data used to calibrate the emerging economy model consists of averages from the sample of emerging economies data and for some parameters data only for Mexico was used.

¹⁰F. Bianchi (2012) finds that the appointment of Volcker marked a change in the conduct of monetary policy.

years and of high volatility periods of 4 years.¹¹ The resulting transition probabilities are $F_{ll} = 0.94$ and $F_{hh} = 0.60$. The value for the low volatility is set to the average of both low volatility periods: $\sigma_l = 0.44$ percent. Then, given the long-run probabilities implied by the duration of each period, high volatility is set to $\sigma_h = 1.20$ percent to match the full-sample 1953-2016 average volatility of 0.55 percent.

For the interest rate process, the Tauchen and Hussey (1991) discretization algorithm was used with 5 grid points, mean interest rate of 0.7 percent, and autocorrelation coefficient 0.479 for the high volatility process and 0.799 for the low volatility process. The autocorrelation coefficients were estimated using the periods identified in Section 2.2.2. Regarding the TFP shock, the autoregressive coefficient and standard deviation were set to commonly used values for small open economies of 0.54 and 2.58 percent respectively (see J. Bianchi, 2011).

Finally, the debt fraction of foreign collateral κ_f is assumed to follow a two-state regime-switching process. The parameter κ_f will take the value of 0 for low-risk periods and 0.05 for high-risk periods following the evidence documented in World Bank (2017). The transition matrix calibration is set to capture the length of a full presidential term in Mexico of 6 years for high-risk periods, and for the low-risk periods, the duration is calibrated such that when there is no expropriation risk the average capital to GDP ratio is equal to the advanced economies average of 2.4. Table 2.1 shows the calibrated parameters.

2.4.2 Quantitative results

This paper explores the role of FDI during Sudden Stop episodes. In particular, the analyzed mechanism has two effects: the direct effect that comes from having a

¹¹The sample used starts at the beginning of a full observed period of low volatility and ends at the end of a full observed period of high volatility.

Table 2.1: Calibrated Parameters

Parameter	Value	Source or Target
Common in the literature		
ν	2	From Mendoza (2010)
ω	1.85	From Mendoza (2010)
δ	8.8%	From Garcia-Verdú (2005)
A	1.0	Normalized value
Matched moments		
β	0.956	Match SS probability of 2.9%
\bar{k}_{EE}	1.90	Match FDI/GDP% = -3.9 for emerging eco.
\bar{k}_{AE}	3.14	Match FDI/GDP% = -0.3 for advanced eco.
α	0.23	Match K/GDP = 2.1 for emerging eco.
κ	0.22	Match Mexico's Debt/GDP ratio -35%
ϕ	8.5	Match s.d.(PI)/s.d(FDI) = 1.85 in emerging eco.
Exogenous process		
\bar{r}	0.7%	Match US average real interest rate data
r	$\{-0.8, 0.0, 0.7, 1.4, 2.3\}$	Discretized using Tauchen and Hussey (1991)
ρ_r	$\{0.799, 0.479\}$	Estimated using low and high volatility periods
σ_l	0.44%	Match average of low volatility periods
σ_h	1.20%	Match average full sample volatility
F_{ll}	0.94	Match duration of low volatility periods
F_{hh}	0.60	Match duration of high volatility periods
ρ	0.58	From J. Bianchi (2011)
σ	2.58%	From J. Bianchi (2011)
κ_f	$\{0, 0.05\}$	World Bank (2017) 5% of expropriation
$F_{ll,\kappa}$	0.92	Match K/GDP = 2.4 when no exp. risk
$F_{hh,\kappa}$	0.83	Match presidential term in Mexico (6 years)

positive probability of expropriation and hence increasing the debt capacity of the economy, and the indirect effect that comes from movements in the FDI account during a crisis that affects the price of capital and hence the market value of all the collateral.

To account for the role of FDI, we compare the results obtained from an emerging economy following the calibration proposed in Section 2.4.1 with the results obtained from the calibration of an advanced economy. To discipline the quantitative results, the advanced economy calibration will defer only in two ways from the emerging economy calibration. First, as noted in Section 2.3, the advanced economy will have a larger stock of domestic capital (this is a proxy to having outflow FDI and hence reduce the net FDI position), and second, following the World Bank (2017), the advanced economy will not be exposed to any expropriation risk. To additionally motivate that advanced economies have no expropriation risk we use the International Country Risk Guide (ICRG) database.¹² In particular, we will use the variable that corresponds to Investment Profile (*inv*) to document any correlation evidence between expropriation risk and FDI in both groups of economies. The *inv* variable takes values from 0 (very high risk) to 12 (very low risk). Column (1) of Table 2.3 shows the results from a descriptive panel regression model that includes as explanatory variables the lag US interest rate level, the lag US interest rate volatility, the interaction of the *inv* variable with both a dummy variable for advanced economies and a dummy variable for emerging economies and country Fixed Effects. From the coefficients of the interaction of the investment profile variable we get two results. First, focusing on the effect of investment risk in advanced economies ($-inv * Dummy Adv$), the regression coefficient is not statistically different from zero suggesting that in fact,

¹²The ICRG database is a well-known source for political and economic risk measures and has been used by Herrera, Ordoñez, and Trebesch (2020) among others.

the expropriation risk is only present in emerging economies. Second, the coefficient for the emerging economies ($-inv * Dummy\ Eme$) is highly significant and negative meaning that more risk decreases the FDI flows into the economy (since the regression is done with $-inv$, higher numbers mean more risk). Hence, as expected, expropriation risk increases the cost of FDI, disincentives multinationals to invest in the domestic economy, and is only present in emerging economies.

After solving both models, we simulated 200,000 periods and dropped the first 10,000 points.¹³ Table 2.2 shows the moments of the simulated data for both classifications of economies. To discipline the results we match Fact 2 and Fact 3 described in Section 2.2 and will use the structural model to quantify the role of the FDI channel in the probability of a Sudden Stop (Fact 1).

Concerning the business cycle moments, the middle section of Table 2.2 shows that the calibrated models are consistent with advanced economies having larger GDP per capita than emerging economies and having more debt-to-income ratios. The model suggests that advanced economies are 11 percent larger than emerging and have 46 percent more debt relative to their GDP than emerging economies. Also, in line with the evidence presented in Section 2.2, the capital outflows distribution for the advanced economy model has thinner tails and a Kurtosis coefficient 1 unit below the coefficient in the emerging economy model.

Finally, with respect to the probability of a Sudden Stop (Fact 1), the model suggests that an emerging economy that increases the outflow FDI and eliminates the expropriation risk would reduce the probability of a Sudden Stop to 1.3 percent.

Figure 2.6 shows the simulated dynamics of the variables of interest during a

¹³We use the *FiPIt* algorithm proposed by Mendoza and Villalvazo (2020). Note that a global solution method is required due to the time-varying volatility in the interest rate and the high nonlinearities that models with occasionally-binding constraints are characterized to show in the policy functions.

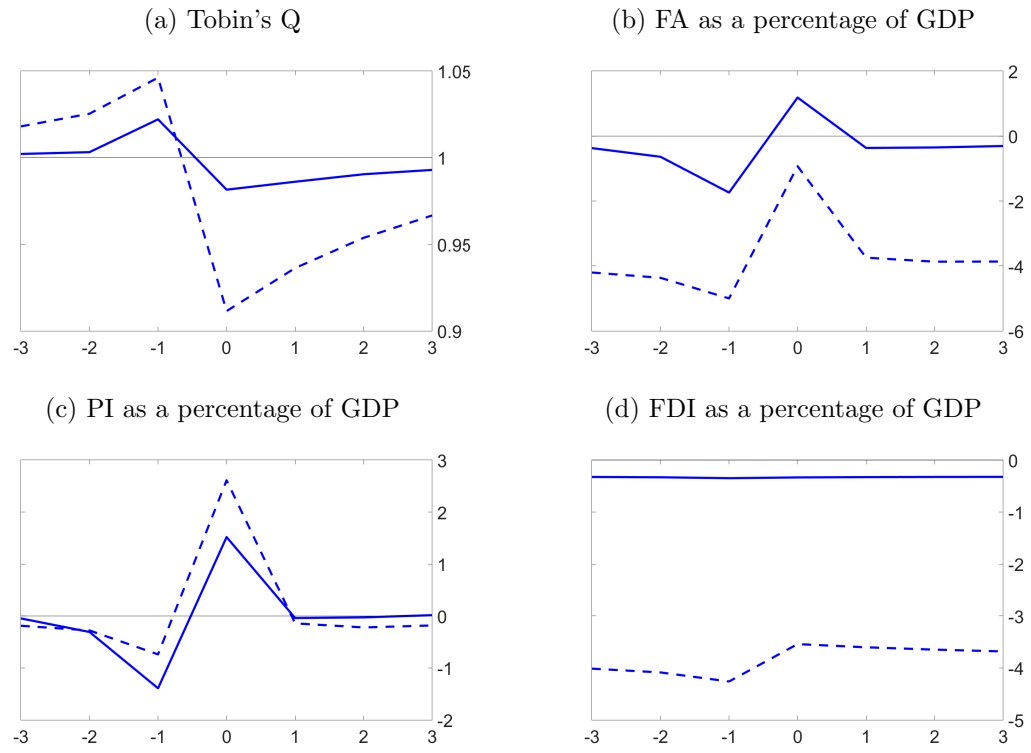
Table 2.2: Simulated Statistics

	Emerging Eco.	Advanced Eco.	
Matched Business Cycles Moments			
Mean FDI / GDP %	-3.9%	-0.3%	Fact 2
Mean Capital / GDP	2.1	2.4	Fact 3
Business Cycle Moments			
Mean GDP (index EE = 100)	100	111	
Mean Debt / GDP %	35%	51%	
Kurtosis of capital outflows	4.7	3.6	Fact 1
Sudden Stops			
Long-run prob. of SS (matched for EE)	2.9%	1.3%	Fact 1

Sudden Stop. With respect to the price of the capital (Tobin's Q), the drop in the emerging economy model is about 8 percent and this drop is 6 percentage points larger than in the advanced economy model. Regarding the Financial Account, advanced economies have smaller deficits in the FA, while emerging economies show a larger contraction in the FA consistent with the data presented in Figure 2.4. This difference is due mainly to the FDI channel since both groups show similar dynamics in the Portfolio flows. Also in line with the data, advanced economies have a larger deficit in the Portfolio flows and register a larger contraction during the Sudden Stop. Finally, there is a large contraction in FDI flows in the emerging economy and no movement in advanced economies also consistent with the evidence presented in Section 2.2.

Finally, in Column (2) of Table 2.3, we compare the results obtained from a descriptive regression using the simulated data from the model with the results obtained from the panel database in Column (1). Since the expropriation risk is only present in the emerging economy model, instead of having the investment risk interaction with a dummy variable for each economy group, we use the time series of the probability of expropriation κ_f to measure the effect of expropriation risk in emerging economies. We can see that the results from the simulated data obtained from the model are

Figure 2.6: Simulated Event Study of a Sudden Stop



Notes: Solid (dashed) lines correspond to advanced (emerging) economy model.

consistent qualitatively with the results from the panel database.

Table 2.3: Descriptive Regression

	<i>Dependent variable: $-FDI / GDP_{i,t}$ %</i>	
	Real Data (1)	Simulated Data (2)
$r_mean_{US,t-1}$	-0.452** (0.183)	-0.030*** (0.003)
$r_vol_{US,t-1}$	0.580 (0.751)	-1.041*** (0.010)
-inv * Dummy Adv	0.430 (0.264)	—
-inv * Dummy Eme	-0.524** (0.249)	—
κ_f	—	-2.718*** (0.018)
Country FE	<i>YES</i>	<i>YES</i>
Observations	1,923	389,997
R ²	0.199	0.629
<i>Notes:</i> *p<0.1; **p<0.05; ***p<0.01		

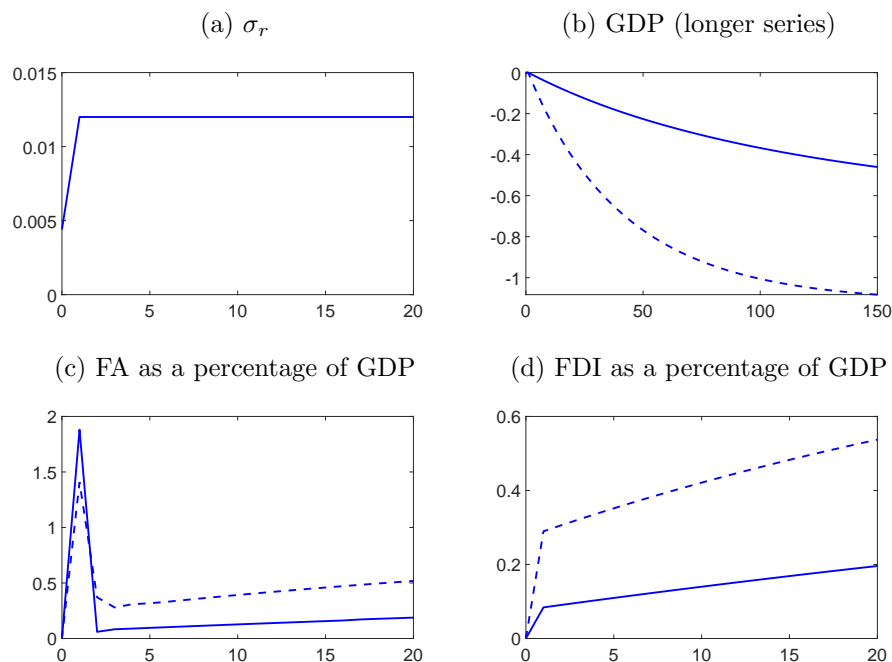
In terms of the signs of the coefficients, the model does a successful job. With simulated data, the regression coefficients with respect to the interest rate level and volatility are negative. An interpretation of this is that as the international interest rate increases, the opportunity cost of FDI investment increases and capital that had previously enter the economy will be reallocated and invested at the international rate. Concerning the volatility coefficient, in line with the results from C. M. Reinhart and V. R. Reinhart (2001), with simulated data the regression coefficient suggests that as the volatility increases FDI decreases while the coefficient using real data is not statistically significant. Finally, the coefficient for investment risk (κ_f in the simulated data) in the emerging economies is highly significant and negative for both real data and simulated data. Suggesting that as investment risk increases, net FDI flows into the domestic economy decrease.

2.4.3 Impulse response analysis

As documented in Fernández-Villaverde et al. (2011), changes in volatility affect real term variables. This paper extends their analysis to study how changes in volatility can cause Sudden Stops and quantifies the effect on the Financial Account flows and the GDP. Figure 2.7 corresponds to a permanent increase in the US interest rate volatility from a high interest rate and mean productivity state. The model suggests that a permanent increase would generate a permanent contraction in the FA/GDP ratio close to 0.5 percent in an emerging and to 0.25 percent in an advanced economy, and a permanent decrease of 1.1 percent in the GDP of emerging and of 0.5 percent in advanced economies. However, in the short run, the outflows of capital are 0.5 percent higher in advanced economies. With respect to a temporary shock, Figure 2.8 shows that a temporary increase of 1 period in the volatility of the US interest rate would generate a Sudden Stop in both types of economies and a contraction in the FA/GDP ratio of close to 2 and 1.5 percent in advanced and emerging economies, respectively. However, this increase in volatility would generate a decrease of only 0.03 percent in emerging economies while a decrease of about 0.005 percent in advanced economies. Hence, the model suggests that in terms of the GDP effect from a temporary increase in the international volatility, advanced economies are more resilient.

To account for the importance of providing certainty to international investors and multinationals, we perform an impulse response analysis after a shock to the probability of expropriation. Figure 2.9 corresponds to a permanent increase in the probability of expropriation from a mean interest rate and mean productivity state. This shock generates a permanent contraction in the FA/GDP ratio of 1.5 percent and a permanent decrease of 3.5 percent in the emerging economy's GDP. However, in the short run, the FA/GDP ratio shows a large contraction of 3 percent. With respect to a temporary shock, Figure 2.10 shows that a temporary increase of 1 period

Figure 2.7: Impulse response analysis after a permanent increase in the international interest rate volatility



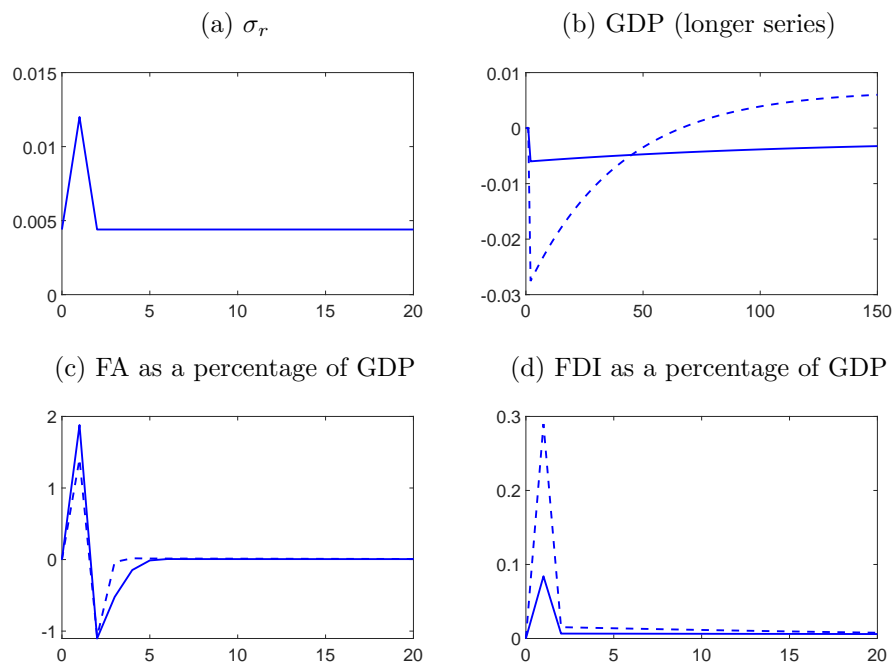
Notes: Solid (dashed) lines correspond to advanced (emerging) economy model.

in the risk of expropriation would cause a large Sudden Stop with a contraction in the FA/GDP ratio of 3 percent. Moreover, this increase in expropriation risk would generate a temporary decrease close to 0.09 percent in GDP.

2.4.4 Anecdotal evidence: episodes of expropriation

To give the previous results some historical context, in this section we present anecdotal evidence of temporary and permanent episodes of increases in the risk of expropriation. For the case of a temporary shock, in Mexico in 1982, 3 months before leaving the office, President Jose Lopez Portillo nationalized the banks. However, by 1984 almost all assets were re-privatized and by 1990 only 18 out of the 58 originally nationalized banks remained (Haber, 2005 and Gruben, McComb, et al., 1997). Figure 2.11.a shows how after the nationalization, FDI/GDP ratio dropped 0.8 percent

Figure 2.8: Impulse response analysis after a temporary increase in the international interest rate volatility

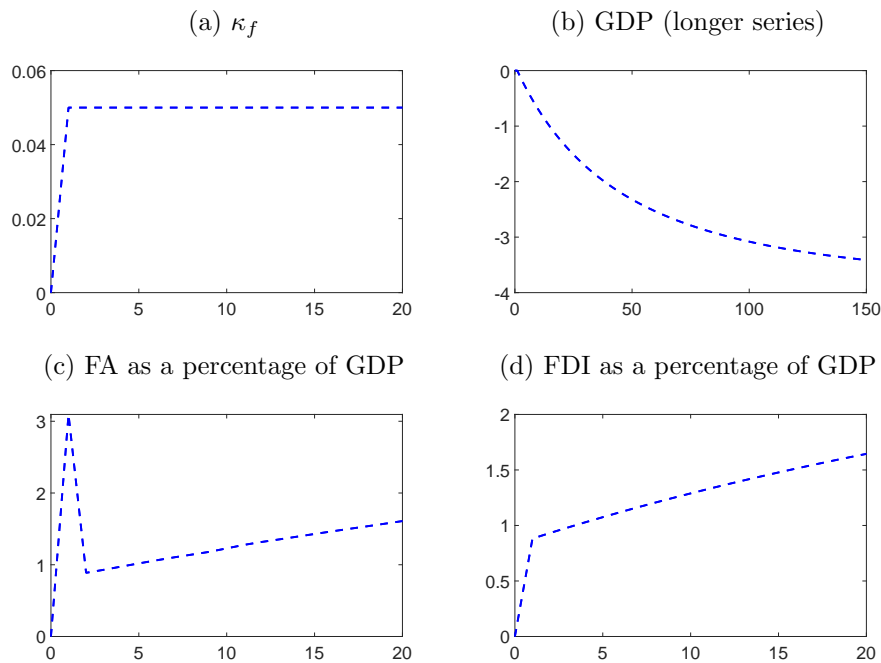


Notes: Solid (dashed) lines correspond to advanced (emerging) economy model.

and the GDP decreased 4.2 percent in 1983. The drop in FDI is similar to the drop obtained by the model as Figure 2.10.d shows. However, it is important to note that the movement in the GDP is larger than the results of the previous section because, among other things, this episode was of an actual privatization and not only an increase in the risk of privatization.

With respect to a permanent shock, in Venezuela in 1998, after Hugo Chavez was elected president the risk of expropriation increased and it was until 2003 when the oil industry was re-nationalized (Weisbrot, Ray, Sandoval, et al., 2009). Figure 2.11.b shows how from 1997 to 2001 the FDI/GDP ratio decreased 1 percent. In this case, the results obtained from the model for a permanent shock in the risk of expropriation (Figure 2.9.b and 2.9.d) are in line with the anecdotal evidence from Venezuela when only the risk of expropriation increased (the GDP decreased by 5

Figure 2.9: Impulse response analysis after a permanent increase in the risk of expropriation



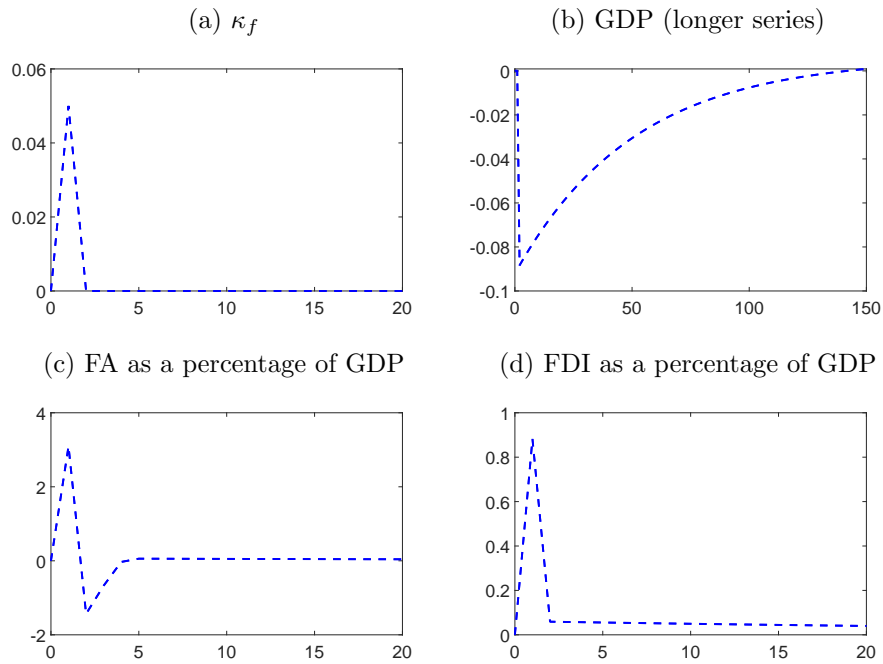
Notes: Dashed lines correspond to emerging economy model.

percent from 1997 to 2001). However, in 2002 and 2003 large expropriations (the oil industry was nationalized) happened in Venezuela and after 2003, the GDP increased dramatically, possibly due to a large increase in oil prices that went from \$30 to \$100 dollars per barrel and also to a lack of credibility in the Venezuelan national accounts.

2.5 Conclusions

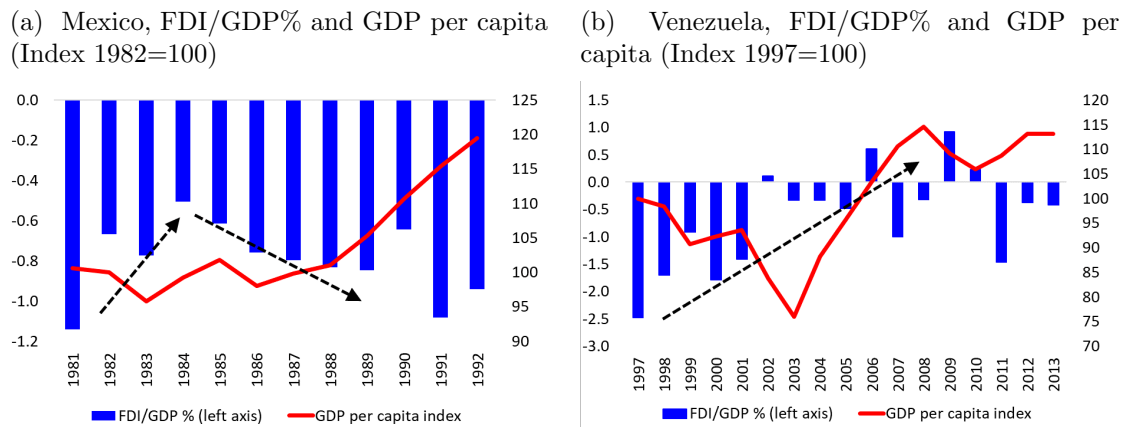
Balance of payment crises, characterized by Sudden Stops, are not a phenomenon exclusive to emerging economies. However, the underlying factors are not necessarily the same; these countries have opened their economies to foreign capital in distinct ways. These differences motivate the study of the components of capital flows in both types of economies to better understand why the probability of having a Sudden Stop in an emerging economy is 20 percent larger than in advanced economies.

Figure 2.10: Impulse response analysis after a temporary increase in the risk of expropriation



Notes: Dashed lines correspond to emerging economy model.

Figure 2.11: Episodes of Expropriations



Source: World Bank WDI.

Decomposing the Financial Account uncovers important differences between advanced and emerging economies in their FDI account. First, advanced economies have on average zero net FDI flows as a percentage of GDP, and second, advanced economies have sufficient FDI outflows that act as a buffer saving during Sudden Stops. To quantify the effect of the FDI channel on the probability of a SS, we propose a standard real business cycle of a small open economy model with an endogenous occasionally-binding constraint, a fixed domestic stock of capital and foreign investment subject to expropriation risk, that generates Sudden Stop crises endogenously.

We calibrate the model using data for a large sample of advanced and emerging economies and find that the FDI channel has a large impact on the probability of a Sudden Stop. In particular, the model's results suggest that on average an emerging economy that increases their capital to GDP ratio and eliminates the expropriation risk would reduce the probability of a Sudden Stop from 2.9 to 1.3 percent and would increase its debt-to-income ratio from 35 percent to 51 percent.

Also, the impulse response analysis suggests that a temporary (permanent) increase in the international interest volatility would lead to a short-run (long-run) decrease of 0.03 (1.1) percent in the GDP in emerging economies. Moreover, in advanced economies, although the movements in the Financial Account are 0.5 percentage points larger than in emerging economies, the effect in the GDP is a third of the magnitude of emerging economies. Regarding the expropriation risk, a temporary (permanent) increase in the expropriation risk would lead to a short-run (long-run) decline of 0.09 (3.5) in the GDP for an emerging economy.

On the policy side, in addition to encouraging a stronger rule of law that would bring certainty to foreign investors (i.e. reduce the risk of expropriation), emerging economies should promote policies that encourage outflow FDI to diversify the capital flows and become more resilient to volatility shocks. This would reduce the probability

and the severity of a Sudden Stop crisis while increasing the debt capacity of the economy.

References

- Albuquerque, Rui (2003). “The composition of international capital flows: risk sharing through foreign direct investment”. In: *Journal of International Economics* 61.2, pp. 353–383.
- Albuquerque, Rui, Norman Loayza, and Luis Servén (2005). “World market integration through the lens of foreign direct investors”. In: *Journal of International Economics* 66.2, pp. 267–295.
- Antras, Pol, Mihir A Desai, and C Fritz Foley (2009). “Multinational firms, FDI flows, and imperfect capital markets”. In: *The Quarterly Journal of Economics* 124.3, pp. 1171–1219.
- Backus, David K, Patrick J Kehoe, and Finn E Kydland (1992). “International real business cycles”. In: *Journal of Political Economy* 100.4, pp. 745–775.
- Baxter, Marianne and Mario J Crucini (Nov. 1995). “Business cycles and the asset structure of foreign trade”. In: *International Economic Review* 36.4, pp. 821–854.
- Bianchi, Francesco (2012). “Evolving monetary/fiscal policy mix in the united states”. In: *American Economic Review* 102.3, pp. 167–72.
- Bianchi, Javier (2011). “Overborrowing and systemic externalities in the business cycle”. In: *American Economic Review* 101.7, pp. 3400–3426.
- Bianchi, Javier and Enrique G Mendoza (2020). “A Fisherian approach to financial crises: Lessons from the Sudden Stops literature”. In: *Review of Economic Dynamics*.

- Calvo, Guillermo A., Alejandro Izquierdo, and Ernesto Talvi (May 2006). “Sudden Stops and Phoenix Miracles in Emerging Markets”. In: *American Economic Review* 96.2, pp. 405–410. DOI: [10.1257/000282806777211856](https://doi.org/10.1257/000282806777211856). URL: <http://www.aeaweb.org/articles?id=10.1257/000282806777211856>.
- Fernández-Villaverde, Jesús et al. (2011). “Risk matters: The real effects of volatility shocks”. In: *American Economic Review* 101.6, pp. 2530–61.
- Garcia-Cicco, Javier, Roberto Pancrazi, and Martin Uribe (2010). “Real business cycles in emerging countries?” In: *American Economic Review* 100.5, pp. 2510–31.
- Garcia-Verdú, Rodrigo (2005). “Factor shares from household survey data”. In: *Banco de México, Working Paper* 5.
- Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman (1988). “Investment, capacity utilization, and the real business cycle”. In: *The American Economic Review*, pp. 402–417.
- Gruben, William C, Robert P McComb, et al. (1997). “Liberalization, privatization, and crash: Mexico’s banking system in the 1990s”. In: *Economic Review-Federal Reserve Bank of Dallas*, pp. 21–30.
- Haber, Stephen (2005). “Mexico’s experiments with bank privatization and liberalization, 1991–2003”. In: *Journal of Banking & Finance* 29.8-9, pp. 2325–2353.
- Hajzler, Christopher (2012). “Expropriation of foreign direct investments: sectoral patterns from 1993 to 2006”. In: *Review of World Economics* 148.1, pp. 119–149.
- Heathcote, Jonathan and Fabrizio Perri (2002). “Financial autarky and international business cycles”. In: *Journal of Monetary Economics* 49.3, pp. 601–627.
- Herrera, Helios, Guillermo Ordoñez, and Christoph Trebesch (2020). “Political booms, financial crises”. In: *Journal of Political Economy* 128.2, pp. 000–000.

- International Monetary Fund (2013). “Balance of Payments and International Investment Position Manual”. In: *Sixth Edition*.
- (2015). “Making Public Investment More Efficient”. In: *Policy Papers*.
- Justiniano, Alejandro and Giorgio E Primiceri (2008). “The time-varying volatility of macroeconomic fluctuations”. In: *American Economic Review* 98.3, pp. 604–41.
- Mendoza, Enrique G (2010). “Sudden stops, financial crises, and leverage”. In: *The American Economic Review* 100.5, pp. 1941–1966.
- Mendoza, Enrique G and Katherine A Smith (2006). “Quantitative implications of a debt-deflation theory of sudden stops and asset prices”. In: *Journal of International Economics* 70.1, pp. 82–114.
- Mendoza, Enrique G and Sergio Villalvazo (2020). “FiPIt: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints”. In: *Review of Economic Dynamics* 37, pp. 81–102.
- Reinhart, Carmen M and Vincent R Reinhart (2001). *What hurts most? G-3 exchange rate or interest rate volatility*. Tech. rep. National Bureau of Economic Research.
- Tauchen, George and Robert Hussey (1991). “Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models”. In: *Econometrica: Journal of the Econometric Society*, pp. 371–396.
- Thomas, Jonathan and Tim Worrall (1994). “Foreign direct investment and the risk of expropriation”. In: *The Review of Economic Studies* 61.1, pp. 81–108.
- Uribe, Martin and Stephanie Schmitt-Grohé (2017). *Open economy macroeconomics*. Princeton University Press.
- Weisbrot, Mark, Rebecca Ray, Luis Sandoval, et al. (2009). “The Chávez administration at 10 years: The economy and social indicators”. In: *Center for economic and policy research*, pp. 1–24.

World Bank (2017). “Global investment competitiveness report 2017-2018 : foreign investor perspectives and policy implications”. In: *World Bank Group*.

2.A Appendix

2.A.1 Description of the data

The panel database consists of 31 high income economies and 75 upper-middle income economies according to the World Bank’s classification. Data on the Financial Account components comes from the IMF Balance of Payments Statistics, GDP comes from the World Bank National Accounts database, capital stocks come from the IMF Investment and Capital Stock Dataset, debt stocks come from the Joint External Debt Hub, and the US interest rate comes from the FRED. The list of countries and Sudden Stop episodes are:

Table 2.4: List of Countries

Albania	Upper-Middle Income	Jordan	Upper-Middle Income
Algeria	Upper-Middle Income	Kazakhstan	Upper-Middle Income
Angola	Upper-Middle Income	Korea, Republic of	Upper-Middle Income
Antigua and Barbuda	Upper-Middle Income	Kuwait	High Income
Argentina	Upper-Middle Income	Latvia	Upper-Middle Income
Australia	High Income	Lebanon	Upper-Middle Income
Austria	High Income	Libya	Upper-Middle Income
Azerbaijan, Republic of	Upper-Middle Income	Lithuania	Upper-Middle Income
Bahamas, The	High Income	China, P.R.: Macao	High Income
Bahrain, Kingdom of	Upper-Middle Income	Macedonia, FYR	Upper-Middle Income
Barbados	Upper-Middle Income	Malaysia	Upper-Middle Income
Belarus	Upper-Middle Income	Maldives	Upper-Middle Income
Belgium	High Income	Malta	Upper-Middle Income
Belize	Upper-Middle Income	Marshall Islands, Republic of	Upper-Middle Income
Bermuda	High Income	Mauritius	Upper-Middle Income
Bosnia and Herzegovina	Upper-Middle Income	Mexico	Upper-Middle Income
Botswana	Upper-Middle Income	Montenegro	Upper-Middle Income
Brazil	Upper-Middle Income	Namibia	Upper-Middle Income
Brunei Darussalam	High Income	Netherlands	High Income
Bulgaria	Upper-Middle Income	New Zealand	High Income
Canada	High Income	Norway	High Income
Chile	Upper-Middle Income	Oman	Upper-Middle Income
China, P.R.: Mainland	Upper-Middle Income	Palau	Upper-Middle Income
Colombia	Upper-Middle Income	Panama	Upper-Middle Income
Costa Rica	Upper-Middle Income	Paraguay	Upper-Middle Income
Croatia	Upper-Middle Income	Peru	Upper-Middle Income
Cyprus	High Income	Poland	Upper-Middle Income
Czech Republic	Upper-Middle Income	Portugal	High Income
Denmark	High Income	Romania	Upper-Middle Income
Dominica	Upper-Middle Income	Russian Federation	Upper-Middle Income
Dominican Republic	Upper-Middle Income	Saudi Arabia	Upper-Middle Income
Ecuador	Upper-Middle Income	Serbia, Republic of	Upper-Middle Income
Equatorial Guinea	Upper-Middle Income	Seychelles	Upper-Middle Income
Estonia	Upper-Middle Income	Singapore	High Income
Fiji	Upper-Middle Income	Slovak Republic	Upper-Middle Income
Finland	High Income	Slovenia	Upper-Middle Income
France	High Income	South Africa	Upper-Middle Income
Gabon	Upper-Middle Income	Spain	High Income
Georgia	Upper-Middle Income	St. Kitts and Nevis	Upper-Middle Income
Germany	High Income	St. Lucia	Upper-Middle Income
Greece	Upper-Middle Income	St. Vincent and the Grenadines	Upper-Middle Income
Grenada	Upper-Middle Income	Suriname	Upper-Middle Income
Guyana	Upper-Middle Income	Sweden	High Income
China, P.R.: Hong Kong	High Income	Switzerland	High Income
Hungary	Upper-Middle Income	Thailand	Upper-Middle Income
Iceland	High Income	Trinidad and Tobago	Upper-Middle Income
Iran, Islamic Republic of	Upper-Middle Income	Turkey	Upper-Middle Income
Iraq	Upper-Middle Income	Tuvalu	Upper-Middle Income
Ireland	High Income	United Kingdom	High Income
Israel	High Income	United States	High Income
Italy	High Income	Uruguay	Upper-Middle Income
Jamaica	Upper-Middle Income	Venezuela, Rep. Bolivariana de	Upper-Middle Income
Japan	High Income		

Table 2.5: List of Sudden Stops

Albania	1995	Israel	2001
Angola	2010	Italy	2012
Antigua and Barbuda	2014	Jamaica	2009
Argentina	2002	Jamaica	2015
Argentina	1990	Kazakhstan	2010
Austria	2006	Korea, Republic of	1998
Bahamas, The	2015	Latvia	2009
Bahrain, Kingdom of	1995	Lebanon	2010
Belgium	2010	Lithuania	2009
Belize	2006	Macedonia, FYR	2009
Bosnia and Herzegovina	2009	Malaysia	1998
Botswana	2005	Mauritius	2013
Brunei Darussalam	2010	Mexico	1995
Bulgaria	2009	Montenegro	2009
China, P.R.: Hong Kong	1999	Namibia	2006
Colombia	1999	Norway	2008
Costa Rica	2009	Oman	2010
Cyprus	2009	Palau	2015
Denmark	2011	Panama	2000
Dominica	2016	Paraguay	1996
Dominican Republic	1991	Poland	1994
Ecuador	1999	Portugal	2011
Estonia	2009	Romania	2009
Fiji	2007	Serbia, Republic of	2009
Georgia	2009	Slovenia	2009
Greece	2012	Spain	2009
Grenada	2004	St. Kitts and Nevis	2012
Grenada	2014	St. Lucia	2009
Guyana	1996	Switzerland	2010
Hungary	2009	Thailand	1998
Hungary	1995	Turkey	1994
Iceland	2009	United States	2009
Ireland	2009	Uruguay	2003

Chapter 3

FiPIt: A Simple, Fast Global Method for Solving Models with Two Endogenous States & Occasionally Binding Constraints

BY ENRIQUE G. MENDOZA[†] AND SERGIO VILLALVAZO MARTÍN[‡]

3.1 Introduction

Important branches of the recent macroeconomics literature study quantitative solutions of models in which constraints are triggered endogenously (i.e. they are “occasionally binding”), as in studies of the zero-lower-bound on interest rates or financial crises triggered by credit constraints. Because these models typically feature non-linear decision rules that lack analytic solutions and capture precautionary savings,

[†]University of Pennsylvania, NBER and PIER.

[‡]University of Pennsylvania.

global solution methods (e.g. time iteration or endogenous grids methods) are the preferable tool for solving them. Global methods are, however, less practical than perturbation methods, because of limitations that make them slow and difficult to implement with widely used software (e.g. Matlab). On the other hand, perturbation methods for solving models with occasionally binding constraints, such as OccBin developed by Guerrieri and Iacoviello (2015) and DynareOBC proposed by Holden (2016), have caveats that limit the scope of the findings that can be derived from using them (see Aruoba et al., 2021, Durdu, Groot, and Mendoza, 2019)

This paper proposes a simple and fast algorithm to obtain the global solution of models with two endogenous states and occasionally binding constraints. This algorithm is denoted as *FiPIt* because it is based on the well-known fixed-point iteration approach to solve systems of transcendental equations. It is easy to implement in a Matlab platform and is significantly faster than the standard time iteration algorithm and several hybrid alternatives. *FiPIt*'s solution strategy builds on the class of time iteration methods that originated in the work of Coleman (1990), who first proposed a global solution method based on policy function iterations of the Euler equation. Since then, various enrichments and modifications of this approach have been developed, in particular the endogenous grids method proposed by Carroll (2006) (see Rendahl, 2015 for a general discussion of these methods and an analysis of their convergence properties). *FiPIt* differs from these methods in that it applies the fixed-point iteration method to solve a model's Euler equations. For instance, in the Sudden Stops model solutions provided as example in this paper, the bonds (capital) Euler equation is used to solve directly for a “new” bonds decision rule (capital pricing function) without the need of a non-linear solver. The capital decision rule is solved for in “exact” form using the models' optimality conditions.

The endogenous grids method also avoids using a non-linear solver, but it does

so by defining alternative state variables so that obtaining analytic solutions of Euler equations for control variables (e.g. consumption, investment) requires irregular interpolation of functions defined over endogenous grids of the original state space. This is innocuous in one-dimensional problems, but in two- and higher-dimensional problems it requires elaborate interpolation methods to tackle the non-rectangular nature of the endogenous grids. In particular, Ludwig and Schön (2018) developed a method using Delaunay interpolation, and showed that it is significantly faster than standard time iteration.¹ Alternatively, Brumm and Grill (2014) proposed a variant of the time iteration method that still uses a non-linear solver but gains speed and accuracy by updating grid nodes to track decision rule kinks using also Delaunay interpolation. In contrast, *FiPIt* retains the original state variables so that standard multi-linear interpolation on regular grids can be used.

We apply the algorithm to solve the model proposed by Mendoza (2010), which is a model of Sudden Stops (financial crises) in a small open economy. This model includes an occasionally binding credit constraint limiting intertemporal debt and working capital not to exceed a fraction of the market value of physical capital (i.e. pledgeable collateral). The results show that, relative to the time iteration method, *FiPIt* reduces execution time by a factor of 2.5 (or 18.1 when solving an RBC variant of the model).² We also found that *FiPIt* continues to perform well for several parameter variations, despite the well-known drawback of fixed-point iteration methods indicating that their convergence is not guaranteed. Execution times for seven parameter variations of the

¹Adjacent points in the endogenous grids do not generally match adjacent nodes in the matrix formed by the original grids. Ludwig and Schön tackled this problem using Delaunay interpolation. They also proposed a hybrid method that uses an exogenous grid for one of the endogenous states and an endogenous grid for the second.

²We used Matlab version R2017a on a Windows 10 laptop with an Intel Core i7-6700HQ 2.60GHz chip, 4 physical cores and 16 GB of RAM. The state space for the Sudden Stops (RBC) model has 72 (80) nodes on foreign assets and 30 on domestic capital. The Sudden Stops (RBC) model solved in 810 (100) seconds, compared with 1,986 (1,808) using the time iteration method.

model were smaller than using time iteration by factors of 2.0 to 18.1. Ludwig and Schön (2018) report reductions by factors of 2.7 to 4.1 using endogenous grids with Delaunay interpolation v. standard time iteration, or 1.8 to 2.5 using their hybrid method v. standard time iteration, when solving a perfect-foresight model of human capital accumulation in a small open economy.³

In addition to the Delaunay interpolation, a second drawback of the endogenous grids method relative to the *FiPIt* method is that it still requires a root-finder in order to determine equilibrium solutions in points of the state space in which occasionally binding constraints bind (see Ludwig and Schön, 2018). *FiPIt* requires a non-linear solver only if the solution of the allocations when the constraint binds cannot be separated from the solution of the multiplier of the constraint. The two are separable in models that feature several widely-used occasionally binding constraints, including standard no-borrowing constraints, maximum debt limits, and constraints on debt-to-income and loan-to-value ratios that depend on endogenous variables. Solving variations of the SS model using these constraints, *FiPIt* reduced execution time relative to the time iteration method by a factor of 13.0 for a loan-to-value-ratio constraint and 17.9 for a maximum debt limit.

There are applications in the literature that solve models using fixed-point iteration algorithms with some features similar to the one we proposed here. Carroll (2011) described and implemented a fixed-point iteration algorithm for solving the workhorse complete-markets RBC model of a closed economy. Boz and Mendoza (2014), Bianchi and Mendoza (2018) and Bianchi, Liu, and Mendoza (2016) solved

³They report faster solution times for each individual scenario than with our algorithm but these are not comparable due to differences in models and hardware. We solve a stochastic model with three shocks, capital accumulation and adjustment costs, and a credit constraint that depends on the model’s two endogenous states and a market price. They solve a deterministic model in which human capital is an accumulable factor produced with an exponential technology and a no-borrowing constraint. We do not have details about the software and hardware they used.

open-economy models with occasionally binding collateral constraints iterating on bond decision rules and/or pricing functions. All these applications considered only one endogenous state variable. Perri and Quadrini (2018) solved a two-country model with two endogenous state variables and a credit constraint resulting from an enforcement friction using Fortran and a state space with 121 points (11 nodes for each state variable). This paper differs from these studies in that we develop an algorithm that solves models with two endogenous states easily and fast in a standard Matlab platform and with a sizable state space including 2,160 points. *FiPIt* can be used in a variety of models with two endogenous states. The choice of functions that are iterated on using the Euler equations can vary across models, and there can be more than one arrangement for the same model.

The rest of the paper proceeds as follows. The next section describes the principles of the algorithm in the simple case of a model of savings with endowment income, and uses this example also to explain how *FiPIt* differs from the time iteration and endogenous grids methods. Section 3.3 describes the Sudden Stops model and provides a step-by-step description of the complete algorithm. Section 3.4 provides quantitative results, evaluates the robustness of the algorithm, and conducts performance comparisons with alternative algorithms, including the standard time iteration method. Section 3.5 presents conclusions. In addition, the Matlab codes and an Appendix that provides a user’s guide to the codes are available online.

3.2 A Fixed-Point Iteration Algorithm for a Simple Savings Model

We describe the principles of the *FiPIt* method using a savings model with stochastic endowment income and an exogenous interest rate. This model is a workhorse of

various branches of the macro literature, including consumption and savings in partial equilibrium, heterogeneous agents models with incomplete markets, and international macro models of the small open economy.

A representative agent chooses consumption and savings plans so as to maximize a standard expected utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}. \quad (3.1)$$

subject to the budget or resource constraint:

$$c_t = e^{z_t} \bar{y} + b_t - q b_{t+1}. \quad (3.2)$$

and a debt limit:

$$b_{t+1} \geq -\varphi. \quad (3.3)$$

In the utility function, $\beta \in (0, 1)$ is the subjective discount factor and $u(\cdot)$ is the period utility function, which can be any standard twice, continuously differentiable and concave utility function, although the CRRA functional form is the one used most often:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma},$$

where σ is the relative risk aversion coefficient. In the resource constraint, $e^{z_t} \bar{y}$ is stochastic income with mean \bar{y} and shocks z_t of exponential support e^{z_t} , b_t are holdings of one-period, non-state-contingent discount bonds traded in a frictionless credit market. In a partial equilibrium model of savings or a model of a small open economy, the real interest rate r is exogenous, so the price of bonds is also exogenous and given by $q \equiv \frac{1}{1+r}$. In a general equilibrium model of heterogeneous agents, the above optimization problem is solved by each individual agent facing idiosyncratic income

uncertainty, and the interest rate is endogenously determined so as to clear the bond market. The *FiPIt* method can be used in all of these models, except that in the heterogeneous agents model we would also need to iterate on the interest rate until the bond market clears. We focus on the small open economy case to simplify the exposition.

If the utility function satisfies the Inada condition and income shocks follow a discrete Markov process or a truncated continuous distribution, the debt limit follows from Aiyagari's Natural Debt Limit: agents never choose optimal plans that leave them exposed to the risk of non-positive consumption, and hence never borrow more than the annuity value of the lowest income realization. Alternatively, agents may face an ad-hoc debt limit tighter than the natural debt limit. Thus, the model includes an occasionally binding constraint, albeit of a simple form: $b_{t+1} \geq -\varphi$.

The Euler equation for bond holdings is

$$u_c(c_t) = (1+r)\beta E_t[u_c(c_{t+1})] + \mu_t, \quad (3.4)$$

where $u_c(c_t)$ is the marginal utility of c_t and μ_t is the multiplier on the debt limit. Note that using the resource constraint to substitute for consumption, the Euler equation can be expressed as:

$$u_c(e^{z_t}\bar{y} + b_t - qb_{t+1}) = (1+r)\beta E_t[u_c(e^{z_{t+1}}\bar{y} + b_{t+1} - qb_{t+2})] + \mu_t. \quad (3.5)$$

A competitive equilibrium for this economy is defined by stochastic sequences $[c_t, b_{t+1}]_{t=0}^{\infty}$ that satisfy equations (3.3) and (3.4) for all t . The economy has a well-defined limiting distribution of (b, y) (i.e. a stochastic steady state) only if $\beta(1+r) < 1$ (see Ljungqvist and Sargent, 2012, Ch. 18). This condition is also a general equilibrium outcome in heterogeneous agents models, because otherwise all agents

would want an infinite amount of bonds, which is inconsistent with market clearing in the market of risk-free bonds.

Since there are no inefficiencies affecting the small open economy (other than the incompleteness of asset markets), the competitive equilibrium can be represented as the solution to the following dynamic programming problem:

$$V(b, z) = \max_{c, b'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \sum_{z'} \pi(z', z) V(b', z') \right\}, \quad (3.6)$$

subject to

$$\begin{aligned} c &= e^z \bar{y} + b - qb' \\ b' &\geq -\varphi \end{aligned}$$

The solution to the above Bellman equation is characterized by a decision rule $b'(b, z)$ and the associated value function $V(b, z)$, and the decision rule together with the Markov process of the shocks induce a joint ergodic (unconditional) distribution of bonds and income $\lambda(b, z)$.

“Euler equation” methods typically solve for $b'(b, z)$ over a discrete state space of (b, z) pairs using the recursive equilibrium conditions that follow from the first-order conditions of the above Bellman equation:

$$c(b, z)^{-\sigma} \geq \beta R \sum_{z'} \pi(z', z) (c(b'(b, z), z'))^{-\sigma} \quad (3.7)$$

$$c(b, z) = e^z \bar{y} + b - qb'(b, z). \quad (3.8)$$

The recursive equilibrium of the model is then defined as the pair of decision rules

$c(b, z), b'(b, z)$ that satisfy these two conditions.

The *FiPIt* method poses a conjecture of the decision rule $\hat{b}'_j(b, z)$ in iteration j , defined over the nodes of discrete grids for b and z . Intermediate values are then found by interpolation. The function $\hat{b}'_j(b, z)$ uses the resource constraint to generate its associated consumption function as $c_j(b, z) = e^z \bar{y} + b - q \hat{b}'_j(b, z)$. Using this consumption function, the above first-order conditions can be combined into an equation that solves for the “new” consumption function:

$$c_{j+1}(b, z) = \left\{ \beta R \sum_{z'} \pi(z', z) \left(c_j(\hat{b}'_j(b, z), z') \right)^{-\sigma} \right\}^{-\frac{1}{\sigma}} \quad (3.9)$$

In the right-hand-side of this Euler equation, we need the value of c_{t+1} , which is obtained by evaluating the consumption function at the $t+1$ -values of the state variables: $c_j(\hat{b}'_j(b, z), z')$. Since $\hat{b}'_j(b, z)$ is defined only on the nodes of the grid of bonds, this consumption function is interpolated over its first argument in order to determine $c_j(\hat{b}'_j(b, z), z')$ (i.e. the value of c_{t+1} implied by the conjectured consumption function). Once this is done, the Euler equation solves directly for a new consumption function $c_{j+1}(b, z)$ without a non-linear solver. Using the resource constraint, this new consumption function yields a new decision rule for bonds $b'_{j+1}(b, z)$, which is re-set to $b'_{j+1}(b, z) = -\varphi$ if $b'_{j+1}(b, z) \leq -\varphi$. Then the decision rule conjecture is updated to $\hat{b}'_{j+1}(b, z)$ as a convex combination of $\hat{b}'_j(b, z)$ and $b'_{j+1}(b, z)$: $\hat{b}'_{j+1}(b, z) = (1 - \rho) \hat{b}'_j(b, z) + \rho b'_{j+1}(b, z)$. The process is repeated until $b'_{j+1}(b, z) = \hat{b}'_j(b, z)$ for all (b, z) in the grids, up to a convergence criterion.

Three points raised by Judd (1998) about fixed-point iteration algorithms like this one are worth recalling. First, using collocation methods instead of solving for a finite state space, the fixed-point iteration method can be represented in a form analogous to the Parameterized Expectations method, because the latter is a fixed-

point iteration method that uses simulation and regression to construct conditional expectations. Second, using $0 < \rho < 1$ ($\rho > 1$) to set the decision rule of the next iteration is useful to address possible instability (slow convergence) of the algorithm. Third, a finite state space may be preferable to collocation methods to define the decision rules depending on whether we expect decision rules to be smooth or to have strong curvature. The latter can be particularly important in models with occasionally binding constraints that depend on endogenous variables, such as credit constraints that depend on collateral prices and yield U-shaped decision rules because of the Fisherian debt-deflation mechanism (see Bianchi and Mendoza, 2018). This will be the case in the model solved in the next section.

Fixed-point iteration differs from the time iteration method because the latter applies the conjectured decision rule $\hat{b}'_j(b, z)$ only to substitute for the term b_{t+2} in the right-hand-side of the Euler equation (3.5), and then uses a non-linear solver to solve the resulting non-linear equation for the optimal choice of b_{t+1} as a function of (z_t, b_t) . Hence, we can think of the fixed-point iteration method as a “proxy time iteration method” that substitutes for the b_{t+1} in the right-hand-side of the Euler equation with a proxy that is defined to be the conjectured decision rule, instead of treating that b_{t+1} term as endogenous.⁴ Fixed-point iteration is also different from the endogenous grids method, because it does not redefine the endogenous state variable and instead solves the problem over the original rectilinear grids (b, z) . Still, fixed-point iteration retains the main computational advantage of the endogenous grids method, which is that the Euler equation is reduced to an equation with an analytic solution for the decision rule, avoiding the need to use non-linear solvers.

⁴From this perspective, it may seem as if the *FiPIt* method solves the “incorrect” Euler equation. Yet, as the paper shows, the solutions satisfy the same equilibrium conditions and are negligibly different from those obtained using standard time iteration. This is because *FiPIt* is essentially an application of the standard fixed-point iteration approach to solve transcendental equations.

3.3 The *FiPIt* Method for Two-Dimensional Models

This section provides a detailed description of the steps that the *FiPIt* method follows to solve a model with two endogenous states and an occasionally binding constraint. The model pertains to a small open economy with two endogenous states, capital (k) and net foreign assets (b), and a credit constraint. If the constraint never binds, the algorithm solves a standard RBC model of a small open economy, and if it binds it solves a model with endogenous financial crises or Sudden Stops.

3.3.1 Model structure and equilibrium conditions

The model is the same as in Mendoza (2010), except that the preferences with endogenous discounting are replaced with standard time-separable expected utility with exogenous discounting at rate β . The economy is inhabited by a representative firm-household with preferences defined over stochastic sequences of consumption c_t and labor supply L_t , for $t = 0, \dots, \infty$, given by:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{(c_t - \frac{L_t^\omega}{\omega})^{1-\sigma}}{1-\sigma} \right] \quad (3.10)$$

The agent chooses sequences of consumption, labor, investment, and holdings of real, one-period international bonds, b_{t+1} (the agent borrows when $b_{t+1} < 0$), so as to maximize the above utility function subject to the following budget and collateral constraints:

$$\begin{aligned} c_t(1 + \tau) + k_{t+1} - (1 - \delta)k_t + \frac{a(k_{t+1} - k_t)^2}{2k_t} = \\ A_t F(k_t, L_t, v_t) - p_t v_t - \phi(R_t - 1)(w_t L_t + p_t v_t) - q_t^b b_{t+1} + b_t, \end{aligned} \quad (3.11)$$

$$q_t^b b_{t+1} - \phi R_t (w_t L_t + p_t v_t) \geq -\kappa q_t k_{t+1}. \quad (3.12)$$

The right-hand-side of the budget constraint is the sum of net profits from production and the resources generated by trading assets abroad. Net profits are equal to gross production minus the cost of imported inputs minus the servicing of foreign working capital loans for labor and imported inputs. Gross output is represented by a constant-returns-to-scale technology, $A_t F(k_t, L_t, v_t) = \exp(\epsilon_t^A) A k_t^\gamma L_t^\alpha v_t^\eta$, that requires capital, k_t , labor and imported inputs, v_t , to produce a tradable good sold at a world-determined price (normalized to unity without loss of generality). TFP is subject to a random shock ϵ_t^A with exponential support around a mean of A . Working capital loans pay for a fraction ϕ of the cost of imported inputs and labor in advance of sales. These loans are obtained from foreign lenders at the beginning of each period and repaid at the end. Lenders charge the world gross real interest rate $R_t = R \exp(\epsilon_t^R)$ on these loans, where ϵ_t^R is an interest rate shock around a mean value R . Imported inputs are purchased at an exogenous relative price in terms of the world's numeraire $p_t = p \exp(\epsilon_t^P)$, where p is the mean price and ϵ_t^P is a shock to the world price of imported inputs (i.e., a terms-of-trade shock). The shocks ϵ_t^A , ϵ_t^R , and ϵ_t^P follow a joint first-order Markov process. The resources generated by trading assets abroad are given by $-q_t^b b_{t+1} + b_t$, where q_t is the price of the international bonds, which satisfies $q_t^b = R_t^{-1}$.

The left-hand-side of the budget constraint is the sum of consumption expenditures, investment and capital adjustment costs. Gross investment is $i_t = k_{t+1} - (1 - \delta)k_t$ and gross investment inclusive of adjustment costs is $\tilde{i}_t = k_{t+1} - (1 - \delta)k_t + \frac{a}{2} \frac{(k_{t+1} - k_t)^2}{k_t}$. Since government expenditures are not included in the model, we include a time-invariant consumption tax τ that is used to calibrate the model to match the

average share of government expenditures in GDP in the data. This is done so that consumption and investment shares in the model can match their data counterparts. Since the tax is constant, it does not distort the savings-consumption margin. The tax does distort labor supply but this distortion is constant over time, since the tax itself is constant.

The credit constraint limits the total debt, which is equal to intertemporal debt plus working capital financing, not to exceed the fraction κ of the market value of the end-of-period capital stock. This is a more complex constraint than borrowing constraints of the class $b_{t+1} \geq -\varphi$, widely used in heterogeneous agents models and also in the algorithm proposed by Ludwig and Schön (2018). Notice that the prices q_t and w_t that appear in this constraint (and the wage in the budget constraint), are endogenous market prices taken as given by the agent when solving its optimization problem. As in Mendoza (2010), the wage rate must be on the labor supply curve (i.e. it must equal the tax-adjusted marginal disutility of labor), which requires $w_t = L^{\omega-1}(1 + \tau)$, and the price of capital must satisfy the optimality condition requiring that $q_t = \frac{\partial \tilde{i}_t}{\partial k_{t+1}}$. With these simplifications noted, the competitive equilibrium of the economy can be represented with the optimization problem of the firm-household, instead of defining separate problems for households and firms. This equilibrium, however, cannot be represented as the solution to a planner's problem formulated as a single Bellman equation, because the planner would internalize the responses of wages and asset prices to its optimal plans, while the representative firm-household does not.

Defining λ_t and μ_t as the future-value multipliers of the budget and collateral constraints respectively, the model's equilibrium conditions in sequential form are:

$$\left(c_t - \frac{L_t^\omega}{\omega}\right)^{-\sigma} = \lambda_t(1 + \tau) \quad (3.13)$$

$$A_t F_{L_t}(k_t, L_t, v_t) = w_t \left(1 + \phi(R_t - 1) + \frac{\mu_t}{\lambda_t} \phi R_t \right) \quad (3.14)$$

$$A_t F_{v_t}(k_t, L_t, v_t) = p_t \left(1 + \phi(R_t - 1) + \frac{\mu_t}{\lambda_t} \phi R_t \right) \quad (3.15)$$

$$\lambda_t = \frac{1}{q_t^b} \beta E_t[\lambda_{t+1}] + \mu_t \quad (3.16)$$

$$\lambda_t = \frac{1}{q_t} \beta E_t \left[\lambda_{t+1} \left(\exp(\epsilon_{t+1}^A) F_{k_{t+1}} - \delta + \frac{a}{2} \frac{(k_{t+2} - k_{t+1})^2}{k_{t+1}^2} + q_{t+1} \right) \right] + \mu_t \kappa \quad (3.17)$$

$$q_t = \frac{\partial \tilde{i}_t}{\partial k_{t+1}} = 1 + a \left(\frac{k_{t+1} - k_t}{k_t} \right) \quad (3.18)$$

$$w_t = L_t^{\omega-1} (1 + \tau) \quad (3.19)$$

$$\begin{aligned} c_t(1 + \tau) + k_{t+1} - (1 - \delta)k_t + \frac{a}{2} \frac{(k_{t+1} - k_t)^2}{k_t} = \\ A_t F(k_t, L_t, v_t) - p_t v_t - \phi(R_t - 1)(L_t^\omega(1 + \tau) + p_t v_t) - q_t^b b_{t+1} + b_t \end{aligned} \quad (3.20)$$

Solving this model with the time iteration method requires solving the Euler equations (3.16) and (3.17) as part of a system of non-linear equations. Given conjectures of the decision rules for capital and bonds, and simplifying using the other equilibrium conditions, the two Euler equations form a two-equation system that yields the “new” decision rules. When the collateral constraint does not bind, these two Euler equations have their standard forms. When the constraint binds, the multiplier μ_t is

an additional endogenous variable and there is an additional equation, which is the constraint holding with equality. The solution can still be reduced to a two-equation system, by using the constraint to substitute for $q_t^b b_{t+1}$ together with the conjectured decision rules so as to obtain a two-equation system in k_{t+1} and μ_t .⁵

Solving with the endogenous grid method requires defining grids for two alternative state variables (s^1, s^2) such that $s_t^1 \equiv q_t^b b_{t+1}$ and $s_t^2 \equiv k_{t+1}/(1 - \delta)$, and then proceeding as in Ludwig and Schön (2018) to first determine the values of (b_{t+1}, k_{t+1}) associated with each (s_t^1, s_t^2) pair, then use the optimality conditions (including the Euler equations) to solve for the contemporaneous controls, particularly (c_t, i_t) , and then use the resource constraint and the definition of gross investment to extract the implied values of the original endogenous states (b_t, k_t) , namely the endogenous grids. When solving for the contemporaneous controls, the optimality conditions form a system of equations that has an analytic solution, thus avoiding the need to use a non-linear solver, but the endogenous grids of (b_{t+1}, k_{t+1}) are irregular, so interpolation of the relevant functions required to obtain the solution of the system is implemented using Delaunay interpolation.⁶ As noted earlier, *FiPIt* does not need either non-linear solvers or interpolation methods for irregular grids.⁷ Standard bi-linear interpolation over rectangular grids still applies.

⁵If the solution implies a value of b_{t+1} lower than the lower bound of the grid of bonds, we set b_{t+1} to that lower bound and solve again the two-equation system for the values of k_{t+1} and μ_{t+1} consistent with that value of b_{t+1} . Hence, the lower bound of the bonds grid is still treated as a constraint of the form $b_{t+1} \geq -\varphi$.

⁶The Ludwig-Schön algorithm still needs to solve a non-linear equation in order to solve for the contemporaneous controls in states in which their no-borrowing constraint binds.

⁷A non-linear equation may need to be solved for in states in which the credit constraint binds, depending on the structure of the constraint (as we explain in Section 3.3.2), but this is separate from the need to solve a two-Euler-equation non-linear system when time iteration is used to solve models with two endogenous states.

3.3.2 Description of the *FiPIt* algorithm

The *FiPIt* method solves the model's equilibrium conditions in recursive form. The model has two endogenous states, b and k , and three exogenous states, using s to denote the triple of exogenous shocks $s \equiv (A, R, p)$, which includes shocks to TFP (A), the world interest rate (R) and the price of imported inputs (p). The recursive equilibrium is defined by a set of recursive functions for allocations $[b'(b, k, s), k'(b, k, s), c(b, k, s), L(b, k, s), v(b, k, s)]$, prices $[w(b, k, s), q(b, k, s)]$ and multipliers $[\lambda(b, k, s), \mu(b, k, s)]$ that satisfy the recursive representation of Equations (3.13)-(3.20), which is provided in the Appendix.

The recursive equilibrium is solved for over a discrete state space, which requires defining discrete grids for (b, k, s) . The grid for the shock triples $s \in \mathbf{S}$ comes from the discretization of the stochastic processes of the model's three shocks. This is typically done using Tauchen's quadrature method. Here we take \mathbf{S} and the associated Markov transition probability matrix from Mendoza (2010), where \mathbf{S} has eight triples (i.e. each shock has two realizations). For the endogenous states, we define grids with M nodes for bonds and N nodes for capital, respectively: $\mathbf{B} = \{b^1 < b^2 < \dots < b^M\}$, $\mathbf{K} = \{k^1 < k^2 < \dots < k^N\}$. The state space has $M \times N \times 8$ elements and is defined by all $(b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$. Once parameter values and the discrete state space are defined, the *FiPIt* algorithm is implemented following the steps described below.

Step 1. Start iteration j with conjectured functions for the price of capital $\hat{q}_j(b, k, s)$, the decision rule for bonds $\hat{b}'_j(b, k, s)$, and the multiplier ratio $\hat{\mu}_j(b, k, s) \equiv \mu_j(b, k, s)/\lambda_j(b, k, s)$. The first iteration can start with $\hat{\mu}_0(b, k, s) = 0$ so that the first pass runs as if it were an RBC model and only cases where the constraint binds pass positive multipliers to the next iteration. The initial functions can be set to $\hat{q}_0(b, k, s) = 1$ and $\hat{b}'_0(b, k, s) = b$, which imply stationary decision rules for capital

and bonds. Note also that this same algorithm can be used to solve a standard RBC model without the occasionally binding constraint, by simply setting κ high enough so that the constraint never binds.

Step 2. Using the recursive equilibrium conditions, compute the iteration- j implied decision rules for capital $k'_j(b, k, s)$, consumption, investment (inclusive of adjustment costs), labor, inputs and output as shown below. Note that, given $\hat{q}_j(b, k, s)$, the capital decision rule has an analytic solution that follows from optimality condition (3.54) (i.e. the capital decision rule has a closed-form solution as a function of the price of capital). The factor allocation rules follow from the conditions equating marginal products with marginal costs, which include factor prices and financing costs. The wages bill wL is replaced with $(1 + \tau)L^\omega$ because of the optimality condition for labor supply. With these arguments in mind, the iteration- j implied decision rules are:

$$k'_j(b, k, s) = \frac{k}{a} [\hat{q}_j(b, k, s) - 1 + a] \quad (3.21)$$

$$\tilde{i}_j(b, k, s) = (k'_j(b, k, s) - k) \left[1 + \frac{a}{2} \left(\frac{k'_j(b, k, s) - k}{k} \right) \right] - \delta k \quad (3.22)$$

$$v_j(b, k, s) = \left\{ \frac{Ak^\gamma \eta^{\frac{\omega-\alpha}{\omega}} \frac{\alpha}{1+\tau} \frac{\alpha}{\omega}}{p^{\frac{\omega-\alpha}{\omega}} [1 + \phi(R-1) + \hat{\mu}_j(b, k, s)\phi R]} \right\}^{\frac{\omega}{\omega(1-\eta)-\alpha}} \quad (3.23)$$

$$L_j(b, k, s) = \left\{ \frac{\alpha}{\eta(1+\tau)} p v_j(b, k, s) \right\}^{\frac{1}{\omega}} \quad (3.24)$$

$$y_j(b, k, s) = Ak^\gamma L_j(b, k, s)^\alpha v_j(b, k, s)^\eta \quad (3.25)$$

Consumption then follows from the resource constraint:

$$(1 + \tau)c_j(b, k, s) = y_j(b, k, s) - pv_j(b, k, s) - \phi(R - 1)[(1 + \tau)L_j(b, k, s)^\omega + pv_j(b, k, s)] - \tilde{i}_j(b, k, s) - \frac{\hat{b}'_j(b, k, s)}{R} + b \quad (3.26)$$

Note that for points where $\hat{\mu}_j(b, k, s) = 0$, factor allocations and output are the same as for an RBC model without credit frictions, which because of the GHH structure of period utility (i.e. the marginal rate of substitution between c and L is independent of c) depend only on (k, s) . We keep them as functions of all three states because when $\hat{\mu}_j(b, k, s) > 0$ factor allocations and output do depend on the three states.

Step 3. Assume the collateral constraint does not bind. This implies that the *new* decision rule for the modified multiplier is $\hat{\mu}_{j+1}(b, k, s) = 0$, and the *new* decision rules for the rest of the endogenous variables are solved using the recursive equilibrium conditions as follows:

3.1 Factor allocations and output again match the expressions corresponding to an RBC model with perfect credit markets:

$$v_{j+1}(b, k, s) = \left\{ \frac{Ak^\gamma \eta^{\frac{\omega-\alpha}{\omega}} \frac{\alpha}{1+\tau} \frac{\alpha}{\omega}}{p^{\frac{\omega-\alpha}{\omega}} [1 + \phi(R - 1)]} \right\}^{\frac{\omega}{\omega(1-\eta)-\alpha}} \quad (3.27)$$

$$L_{j+1}(b, k, s) = \left\{ \frac{\alpha}{\eta(1 + \tau)} pv_{j+1}(b, k, s) \right\}^{\frac{1}{\omega}} \quad (3.28)$$

$$y_{j+1}(b, k, s) = Ak^\gamma L_{j+1}(b, k, s)^\alpha v_{j+1}(b, k, s)^\eta \quad (3.29)$$

3.2 Solve for c_{j+1} by applying the fixed-point iteration method to the Euler equation for bonds. The iteration- j conjectures for capital and bonds are used everywhere in the right-hand-side of this Euler equation, so that we obtain an analytic

solution for c_{j+1} . Keep track of the subscripts denoting which function is used in each term:

$$c_{j+1}(b, k, s) = \left\{ \beta RE \left[\left(c_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s') - \frac{L_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')^\omega}{\omega} \right)^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}} + \frac{L_{j+1}(b, k, s)^\omega}{\omega} \quad (3.30)$$

In the above expression, the functions $c_j(b, k, s)$ and $L_j(b, k, s)$ are defined only at the nodes of $\mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$, but since the values of $\hat{b}'_j(b, k, s)$ and $k'_j(b, k, s)$ generally do not match node grids in \mathbf{B} and \mathbf{K} , respectively, $c_j(\cdot)$ and $L_j(\cdot)$ are interpolated over their first two arguments to determine $c_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')$ and $L_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')$. Standard bi-linear interpolation is applied. Use extrapolation if $k'_j(b, k, s)$ is below (above) k^1 (k^N) and also if $\hat{b}'_j(b, k, s)$ is above b^M , but for $\hat{b}'_j(b, k, s) < b^1$ evaluate the functions at b^1 , because the lower bound on bonds represents an ad-hoc debt limit commonly used for calibration of the model to the data (see Durdu, Groot, and Mendoza, 2019). Note also that, because of the fractional exponent (since typically $\sigma > 1$) the above equation solves only if $c_j(\cdot) - \frac{L_j(\cdot)^\omega}{\omega} > 0$, but if this is true for the consumption and labor decision rules implied by the initial conjectures set for the first iteration ($c_0(\cdot), L_0(\cdot)$), it will also be true at any iteration $j > 0$.

3.3 Solve for $b'_{j+1}(b, k, s)$ using the resource constraint:

$$b'_{j+1}(b, k, s) = R\{y_{j+1}(b, k, s) - pv_{j+1}(b, k, s) - \phi(R - 1)[(1 + \tau)L_{j+1}(b, k, s)^\omega + pv_{j+1}(b, k, s)] - \tilde{i}_j(b, k, s) - (1 + \tau)c_{j+1}(b, k, s) + b\} \quad (3.31)$$

3.4 Evaluate if the collateral constraint binds. If:

$$\frac{b'_{j+1}(b, k, s)}{R} - \phi R [(1 + \tau)L_{j+1}(b, k, s)^\omega + pv_{j+1}(b, k, s)] + \kappa \hat{q}_j(b, k, s)k'_j(b, k, s) \geq 0, \quad (3.32)$$

the constraint does not bind at the point (b, k, s) , the functions with $j + 1$ subscripts are saved, and skip to **Step 5**. Otherwise, the constraint binds at this point, the functions with $j + 1$ subscripts are discarded and move to **Step 4**.

Step 4. Solve for *new* decision rules when the collateral constraint binds. Since $\hat{q}_j(b, k, s)$ has not changed, we use the same iteration- j implied decision rule for capital $k'_j(b, k, s) = \frac{k}{a} [\hat{q}_j(b, k, s) - 1 + a]$ and the same function $\tilde{i}_j(b, k, s)$ as before. This is the most computationally intensive step, because it solves a non-linear simultaneous equations system to determine $L_{j+1}(b, k, s)$, $v_{j+1}(b, k, s)$, $c_{j+1}(b, k, s)$, $b'_{j+1}(b, k, s)$, $\tilde{\mu}_{j+1}(b, k, s)$. The five equations in the system are the two optimality conditions for factor allocations, the Euler equation for bonds (with the $\tilde{\mu}$ terms), the credit constraint holding with equality, and the resource constraint. To make the solution more tractable, we express $L_{j+1}(b, k, s)$, $v_{j+1}(b, k, s)$, $c_{j+1}(b, k, s)$, $b'_{j+1}(b, k, s)$ as functions of $\tilde{\mu}(b, k, s)$, and use the results to reduce the system to a single non-linear equation in $\tilde{\mu}(b, k, s)$. In the simplified system, factor allocations, consumption and bonds are functions denoted $L_{j+1}(b, k, s, \tilde{\mu})$, $v_{j+1}(b, k, s, \tilde{\mu})$, $c_{j+1}(b, k, s, \tilde{\mu})$, $b'_{j+1}(b, k, s, \tilde{\mu})$, but to make the notation simpler we write them as depending on $\tilde{\mu}$ only (still, keep in mind the set of equations needs to be solved for each (b, k, s) for which the constraint was

found to be binding in step **3.4**):

$$v(\tilde{\mu}) = \left\{ \frac{Ak^\gamma \eta^{\frac{\omega-\alpha}{\omega}} \frac{\alpha}{1+\tau} \frac{\alpha}{\omega}}{p^{\frac{\omega-\alpha}{\omega}} [1 + \phi(R-1) + \tilde{\mu}\phi R]} \right\}^{\frac{\omega}{\omega(1-\eta)-\alpha}} \quad (3.33)$$

$$L(\tilde{\mu}) = \left\{ \frac{\alpha}{\eta(1+\tau)} pv(\tilde{\mu}) \right\}^{\frac{1}{\varepsilon}} \quad (3.34)$$

$$\frac{b'(\tilde{\mu})}{R} = -\kappa \hat{q}_j k'_j + \phi R pv(\tilde{\mu}) \left[1 + \frac{\alpha}{\eta} \right] \quad (3.35)$$

$$(1+\tau)c(\tilde{\mu}) = Ak^\gamma L(\tilde{\mu})^\alpha v(\tilde{\mu})^\eta - pv(\tilde{\mu}) - \phi(R-1)pv(\tilde{\mu}) \left[1 + \frac{\alpha}{\eta} \right] - \tilde{i}_j - \frac{b'(\tilde{\mu})}{R} + b \quad (3.36)$$

The equations for labor and inputs follow from combining the borrowing constraint with the optimality conditions equating marginal products with marginal costs, including the $\tilde{\mu}$ terms. They are the same equations used in **Step 2**, but now we need to find the value of $\tilde{\mu}_{j+1}$ that solves them, instead of taking as given $\tilde{\mu}_j$.

In addition to Equations (3.33)-(3.36), the solution for $\tilde{\mu}_{j+1}(b, k, s)$ must also satisfy the Euler equation for bonds, which can be written as:

$$\tilde{\mu}_{j+1}(b, k, s) = 1 - \frac{\beta RE \left[\left(c_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s') - \frac{L_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')^\omega}{\omega} \right)^{-\sigma} \right]}{\left(c(\tilde{\mu}_{j+1}(b, k, s)) - \frac{L(\tilde{\mu}_{j+1}(b, k, s))^\omega}{\omega} \right)^{-\sigma}} \quad (3.37)$$

Notice the numerator in the second term in the right-hand-side still applies fixed-point iteration by computing expected marginal utility using j-dated functions only. The values of $c_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')$ and $L_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')$ are again determined by bi-linear interpolation.

Algebraic manipulation of Equations (3.33)-(3.37) reduces to this non-linear equation in $\tilde{\mu}_{j+1}(\cdot)$

$$\begin{aligned}
(1 - \tilde{\mu}_{j+1}(\cdot)) & \left\{ C_1^{\frac{\omega}{(1-\eta)\omega-\alpha}} \left[\frac{\alpha}{1 + \phi(R-1) + \tilde{\mu}_{j+1}(\cdot)\phi R} \right]^{\frac{\eta\omega+\alpha}{(1-\eta)\omega-\alpha}} - \left[\frac{\alpha C_1}{1 + \phi(R-1) + \tilde{\mu}_{j+1}(\cdot)\phi R} \right]^{\frac{\omega}{(1-\eta)\omega-\alpha}} C_2 \right. \\
& \left. - \left(\frac{\tilde{i}_j(\cdot) - \kappa \hat{q}_j(\cdot) k'_j(\cdot) - b}{1 + \tau} \right)^{-\sigma} \right\} = \beta RE \left[\left(c_j(\hat{b}'_j(\cdot), k'_j(\cdot), s') - \frac{L_j(\hat{b}'_j(\cdot), k'_j(\cdot), s')^\omega}{\omega} \right)^{-\sigma} \right] \quad (3.38)
\end{aligned}$$

where:

$$C_1 \equiv \left(\frac{1}{1 + \tau} \right)^{1-\eta} A k^\gamma \left(\frac{\eta}{\alpha p} \right)^\eta \quad (3.39)$$

$$C_2 \equiv \frac{1}{\omega} + \frac{\eta}{\alpha} + \phi \left(1 + \frac{\eta}{\alpha} \right) (2R - 1) \quad (3.40)$$

Note again that, because of the fractional exponent in the right-hand-side of (3.38), the equation solves only if $c_j(\cdot) - \frac{L_j(\cdot)^\omega}{\omega} > 0$. Since the first iteration starts with $\tilde{\mu}_0(\cdot) = 0$, any state that yields a binding credit constraint in the first iteration will solve for $\tilde{\mu}_1(\cdot)$ as long as the same condition required for the unconstrained consumption function (Equation 3.30) to solve in the first iteration holds, namely that $c_0(\cdot) - \frac{L_0(\cdot)^\omega}{\omega} > 0$ for the decision rules implied by the initial conjectures set for the first iteration. Moreover, since when the constraint binds it must be true that $0 < \tilde{\mu} < 1$, it follows from Equation 3.37 that $c_j(\cdot) - \frac{L_j(\cdot)^\omega}{\omega} > 0$ will hold for any iteration $j > 0$. Once $\tilde{\mu}_{j+1}(b, k, s)$ is solved, the functions $v_{j+1}(b, k, s)$, $L_{j+1}(b, k, s)$, $b'_{j+1}(b, k, s)$, $c_{j+1}(b, k, s)$ are determined using Equations (3.33)-(3.36), but replacing $\tilde{\mu}$ with $\tilde{\mu}_{j+1}(b, k, s)$. The functions with $j + 1$ subscripts are saved, and we move to **Step 5**.

It is important to note that, depending on the structure of the occasionally binding constraint, if $\tilde{\mu}$ can be solved for separately after solving for the allocations, **Step 4** is much easier because *FiPIt* does not require a non-linear solver *anywhere*. For example, if working capital is not in the credit constraint, we can set $b'_{j+1}(b, k, s)/R = -\kappa \hat{q}_j(b, k, s) k'_j(b, k, s)$, and this can be used to determine $c_{j+1}(b, k, s)$

directly from the resource constraint. The implied value of $\tilde{\mu}_{j+1}(b, k, s)$ can then be solved for from the bonds Euler equation. The same applies for a credit constraint set to a constant value, as in Ludwig and Schön (2018), where they used $b_{t+1} \geq 0$. Hence, *FiPIt* can solve models with a large class of occasionally binding constraints without using a non-linear solver at any point, whereas the Ludwig-Schön algorithm needs both the Delaunay interpolation and a non-linear solver when the constraint binds.

Step 5. Return to **Step 3** and repeat $\forall(b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$. This is necessary before proceeding to compute a new asset pricing function, because the complete set of $j+1$ -dated functions is required.

Step 6. Compute the *new* pricing function $q_{j+1}(b, k, s)$. We describe two ways of doing this:

6.1 The *FiPIt* algorithm proceeds in a manner analogous to fixed-point iteration on the Euler equation for bonds, by applying the new decision rules for $c_{j+1}(\cdot)$, $L_{j+1}(\cdot)$, $b'_{j+1}(\cdot)$, $\tilde{\mu}_{j+1}(\cdot)$ to the Euler equation for capital and solving it so as to obtain the following analytic solution for $q_{j+1}(b, k, s)$:

$$q_{j+1}(b, k, s) = \frac{\beta E_t \left[\left(c_{j+1}(b'_{j+1}(\cdot), k'_j(\cdot), s') - \frac{L_{j+1}(b'_{j+1}(\cdot), k'_j(\cdot), s')^\omega}{\omega} \right)^{-\sigma} [d'(\cdot) + \hat{q}_j(b'_{j+1}(\cdot), k'_j(\cdot), s')] \right]}{\left(c_{j+1}(\cdot) - \frac{L_{j+1}(\cdot)^\omega}{\omega} \right)^{-\sigma} (1 - \kappa \tilde{\mu}_{j+1}(\cdot))} \quad (3.41)$$

where

$$d' (b'_{j+1}(\cdot), k'_j(\cdot), s') = \gamma A' k'_j(\cdot)^{\gamma-1} L_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')^\alpha v_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')^\eta - \delta + \frac{a}{2} \frac{(k'_j(b'_{j+1}(\cdot), k'_j(\cdot), s') - k'_j(\cdot))^2}{k'_j(\cdot)^2}$$

The asset price used in the right-hand-side of (3.41) is the conjecture set in **Step 1**. Since all the functions in the right-hand-side are known, the equation solves directly for $q_{j+1}(b, k, s)$. The values of $c_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')$, $L_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')$ and $\hat{q}_j (b'_{j+1}(\cdot), k'_j(\cdot), s')$ are determined by bi-linear interpolation. The value of the dividends function $d'(\cdot)$ is obtained by applying bi-linear interpolation to evaluate $L_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')$ and $v_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')$ in the marginal product of capital and $k'_j(b'_{j+1}(\cdot), k'_j(\cdot), s')$ in the adjustment cost term. Notice that the decision rule for bonds that sets the value of b_{t+1} at which all these functions are interpolated is a $j+1$ -indexed function, not the j -indexed function used in Steps **3** and **4**, but over the capital dimension we are still using the j -indexed decision rule.

6.2 A variant of the algorithm labeled *Fixed-Point Iteration with Forward Solution (FPIFS)* solves for the new price conjecture by iterating to convergence on the capital Euler equation (i.e. it uses the forward solution of the asset price). Index the iterations on this equation with superscript z , the iterations solve this functional equation problem, always using the $j + 1$ -dated functions and the multiplier $\tilde{\mu}_{j+1}(\cdot)$ obtained in Steps **3** to **5**:

$$q^{z+1}(b, k, s) = \frac{\beta E_t \left[\left(c_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s') - \frac{L_{j+1}(b'_{j+1}(\cdot), k'_j(\cdot), s')^\omega}{\omega} \right)^{-\sigma} [d'(\cdot) + q^z (b'_{j+1}(\cdot), k'_j(\cdot), s')] \right]}{\left(c_{j+1}(\cdot) - \frac{L_{j+1}(\cdot)^\omega}{\omega} \right)^{-\sigma} (1 - \kappa \tilde{\mu}_{j+1}(\cdot))} \quad (3.42)$$

Iterate until $||q^{z+1}(b, k, s) - q^z(b, k, s)|| \leq \varepsilon^q$ for small ε^q , and if the result converges, the final result sets $q_{j+1}(b, k, s)$. The values of $c_{j+1}(b'_{j+1}(\cdot), k'_j(\cdot), s')$, $L_{j+1}(b'_{j+1}(\cdot), k'_j(\cdot), s')$, $d'(\cdot)$ and $q^z(b'_{j+1}(\cdot), k'_j(\cdot), s')$ are determined using bi-linear interpolation as in step **6.1**.

Step 7. Check the convergence of the conjectured functions. Convergence requires that for small ε^f the following conditions are satisfied $\forall (b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$:

$$|q_{j+1}(b, k, s) - \hat{q}_j(b, k, s)| \leq \varepsilon^f \quad (3.43)$$

$$|b'_{j+1}(b, k, s) - \hat{b}'_j(b, k, s)| \leq \varepsilon^f \quad (3.44)$$

$$|\tilde{\mu}_{j+1}(b, k, s) - \hat{\mu}_j(b, k, s)| \leq \varepsilon^f \quad (3.45)$$

If these conditions hold, the recursive competitive equilibrium has been solved. The level of the multiplier on the credit constraint can then be solved for as follows:

$$\mu_{j+1}(b, k, s) = \tilde{\mu}_{j+1}(b, k, s) \left(c_{j+1}(b, k, s) - \frac{L_{j+1}(b, k, s)^\omega}{\omega} \right)^{-\sigma} \quad (3.46)$$

The accuracy of the solution can then be evaluated by verifying that the equilibrium conditions hold, including computations of the maximum and average absolute values of the errors in the Euler equations of k and b .

If any of the three convergence conditions fails, update the conjectured functions using a convex combination of the last conjectures and the new functions to dampen possible overshooting or speed up convergence. This is conventional practice because there is no guarantee that fixed-point iteration algorithms converge, but when they diverge it is generally because they overshoot the true solution. Hence, the new

conjectures are set as:

$$\hat{x}_{j+1}(b, k, s) = (1 - \rho^x)\hat{x}_j(b, k, s) + \rho^x x_{j+1}(b, k, s) \quad (3.47)$$

for $x = [q, b', \tilde{\mu}]$ and some $0 \leq \rho^x$. Notice that $\hat{x}_j(b, k, s)$ in the right-hand-side of this expression represents the initial conjectures that were used in the current iteration, while $\hat{x}_{j+1}(b, k, s)$ in the left-hand-side denotes the new conjectures for the next iteration. Use $0 < \rho^x < 1$ ($\rho^x > 1$) for the particular function $x(\cdot)$ that is not converging (converging too slowly). Return to **Step 2**, setting $\hat{x}_j(b, k, s) = \hat{x}_{j+1}(b, k, s)$, and repeat until convergence is attained.

3.4 Application to Sudden Stops Model

This section examines solutions of the Sudden Stops model obtained with a set of Matlab programs we developed to implement the *FiPIt* algorithm. The Matlab codes and an Appendix that explains how the codes execute each of the algorithm steps are available online. All the computations were made using Matlab R2017a on a Windows 10 laptop with an Intel Core i7-6700HQ 2.60GHz four-core chip and 16GB of RAM.

The model's parameter values are taken from Mendoza (2010) and listed in Table 3.9, including the same baseline value for the collateral coefficient ($\kappa = 0.2$). We also use the same Markov process for the model's three shocks. The only difference, as mentioned earlier, is that instead of using preferences with an endogenous rate of time preference we use standard time-separable expected utility with constant discounting, setting the subjective discount factor at $\beta = 0.92$.

The state space consists of evenly-spaced grids with 72 nodes for bonds and 30

nodes for capital. \mathbf{K} spans the [654.5, 885.5] interval and \mathbf{B} spans the [-188.6, 800] interval. Solving with larger grids increases sharply execution time and produces negligibly different results, while solving with smaller grids is faster but yields inaccurate results. The Markov process of the shocks has two realizations for each shock and their values together with the associated 8×8 transition probability matrix approximate the variability, autocorrelation, and contemporaneous correlation of TFP, interest rates and the price of imported inputs in the data (see Mendoza, 2010 for details).

To assess the performance of the *FiPIt* algorithm, we computed solutions using *FiPIt* and the *FPIFS* variant, as well as solutions from three other algorithms: *TIFS* replaces the fixed-point iteration solution of the bonds decision rule with a standard time iteration solution that uses a non-linear solver, and solves for the price of capital using the forward solution of the capital Euler equation; *TIFPI* uses again standard time iteration for the bonds decision rule, but solves for the price of capital using the fixed-point iteration approach; and *FTI* is the full time iteration solution in which the Euler equations for capital and bonds are solved as a non-linear equation system. In all these solutions except *FTI*, we found faster convergence by setting the dampening parameters for updating the conjectured functions to 0.3 for the price of capital (0.25 for a scenario with 60 capital nodes) and 1 for bonds and $\tilde{\mu}$. For *FTI* solutions, we kept $\rho^x = 1$ for all three functions, and confirmed that these produces convergence in the smallest number of iterations.

3.4.1 Comparison of Results & Performance Metrics

Table 3.2 reports long-run moments of the main macro aggregates and performance statistics of the algorithm for the following seven solution scenarios: Columns (1) and (2) are *FiPIt* solutions with capital grids of 60 and 30 nodes respectively, (3) is the

Table 3.1: Calibrated Parameter Values

Parameter	Value
σ risk aversion coefficient	2.0
ω labor elasticity coefficient	1.8461
β discount factor	0.92
a capital adjustment cost	2.75
ϕ working capital parameter	0.2579
δ depreciation rate	0.088
α labor share	0.59
η imported inputs share	0.10
γ capital share of income	0.31
τ tax on consumption	0.17
A average TFP	6.982
κ collateral coefficient	0.20

TFIS solution, (4) is the *FPIFS* solution, (5) is the *TIFPI* solution, (6) is the *FTI* solution and Column (7) shows the results from Mendoza (2010) for reference.

The comparison of Cols. (1) and (2) shows that solving using *FiPIt* with the smaller capital grid has nearly no effect on the results but reduces execution time by a factor of 2.5. The moments reported in Columns (2) to (6) are very similar, and in fact identical up to one or two decimals. Hence, all of the four solution algorithms we tried yield effectively the same results. The results from Mendoza (2010) in Column (7) are qualitatively similar in terms of ranking of volatilities and signs and ranking of correlations and autocorrelations, but quantitatively show more differences. These are due to the different discount factors (Mendoza used endogenous discounting) and the different solution methods (Mendoza solved forcing decision rules to be on the nodes of the grids of bonds and capital, instead of using interpolation, and used value function iteration on a quasi planner's problem with w and q restricted to satisfy the labor supply and investment optimality conditions). The one item that differs sharply is the probability of Sudden Stops, which is about 2.0 percent in Cols. (1)-(6) v. 3.3

percent in Mendoza’s paper.⁸ This is due to the approximately-continuous decision rules obtained using interpolation in our solutions v. decision rules forced to be on grid nodes in Mendoza’s solution. This makes our estimates of the frequency with which $\mu(b, k, s) > 0$, and of the trade balance adjustment implied by the associated $b'(b, k, s)$ in those states, more accurate. In all of our results, the long-run probability of states with $\mu > 0$ is about 2.6 percent, but 23 percent of these states do not yield a sufficiently large increase in the trade balance to classify as a Sudden Stop.

The performance metrics for Columns (1)-(6) reported in panel (b) of Table 3.2 show that all the solutions have similar accuracy, with small maximum and average absolute-value errors in the Euler equations for bonds and capital. The *FTI* solution yields larger errors, but still this makes little difference in the statistical moments it produces relative to those produced by the other solutions.

In terms of execution time, the *FiPIt* method in Col. (2) dominates the other solution methods by large margins.⁹ The absolute speeds will vary widely with hardware and software configurations, but the relative speeds are likely to vary less and the ranking across methods based on this criterion is unlikely to change. Comparing speeds relative to *FiPIt*, which took 810 seconds to run, the second fastest method is *FPIFS* in Col. (4), which took 20 percent longer solve. This algorithm only differs from *FiPIt* in that it solves for the price of capital by solving forward the capital Euler equation. The slowest methods are the three that use time iteration (i.e. a non-linear solver) for at least one Euler equation. In Cols. (3) and (5) the bonds decision rule is solved with the time iteration method, but the price of capital is solved using the forward solution in Col. (3) v. fixed-point iteration in Col. (5). This

⁸We applied the same definition of Sudden Stops: coordinates (b, k, s) in which the collateral constraint binds and the trade balance-GDP ratio is at least 2 percentage points above what the RBC model yields.

⁹*FiPIt* has even lower relative execution times than the other methods when solving the RBC model, because it avoids using the non-linear solver completely (see Section 3.4.2 for details).

makes little difference in execution time, as they take 4.6 and 5.1 times longer than the *FiPIt* solution in Col. (2), respectively. Interestingly, the standard *FTI* method in Col. (6), which solves the simultaneous non-linear Euler equations for bonds and capital, is significantly *faster* than the methods used in Cols. (3) and (5), indicating that solving only one non-linear Euler equation instead of two does not guarantee a faster algorithm. Still, the *FTI* execution time exceeds that of the *FiPIt* solution by a factor of 2.5!

The *FTI* solution is faster than the ones in Cols. (3) and (5) because time iteration takes advantage of the contraction mapping properties of the two non-linear Euler equations by solving them simultaneously while fixed-point iteration methods do not. Intuitively, every iteration with *FTI* tends to generate relatively more accurate outcomes, and hence attains convergence in 94 iterations. The algorithms in Cols. (3) and (5) take more than twice as many iterations (190 iterations for *TIFS* and 207 for *TIFPI*), and still in each they have to use a root finder because they solve for the bonds decision rule using time iteration. The *FiPIt* method converges in a similar number of iterations (196) as these two methods, but goes through each iteration much faster because it avoids using non-linear solvers when the constraint does not bind, overcoming the drawback of not taking advantage of the contraction mapping properties of the Euler equations, and this makes it the fastest method.¹⁰ *FPIFS* in Col. (4) is the second fastest for a similar reason, and it is slower than *FiPIt* because solving the price of capital with the forward solution is slower than with fixed-point iteration.

In addition to comparing Euler equation errors, we also compared the recursive

¹⁰This suggests that *FiPIt* can be again much faster than *FTI* in applications in which, as explained in Section 3.3, the structure of the occasionally binding constraint is such that *FiPIt* does not need a root-finder in states in which the constraint binds (e.g. $q_t^b b_{t+1} \geq -\kappa q_t k_{t+1}$, $q_t^b b_{t+1} \geq -\varphi$). We show results for a case like this in Section 3.4.2.

Table 3.2: Long-run Moments & Performance Metrics: Sudden Stops Model ($\kappa = 0.2$)

	(1) FiPIt-large grid	(2) FiPIt	(3) TIFS	(4) FPIFS	(5) TIFPI	(6) FTI	(7) Mendoza (2010)
<i>(a) Long-run moments</i>							
<u>Mean</u>							
<i>gdp</i>	393.629	393.619	393.626	393.618	393.626	393.549	388.339
<i>c</i>	273.910	274.123	274.074	274.124	274.073	274.011	267.857
<i>i</i>	67.482	67.481	67.484	67.481	67.484	67.459	65.802
<i>nx/gdp</i>	0.016	0.015	0.015	0.015	0.015	0.015	0.024
<i>k</i>	765.191	765.171	765.202	765.170	765.202	764.922	747.709
<i>b/gdp</i>	0.007	0.015	0.013	0.015	0.013	0.012	-0.104
<i>q</i>	1.000	1.000	1.000	1.000	1.000	1.000	1.000
<i>leverage ratio</i>	-0.106	-0.102	-0.103	-0.102	-0.103	-0.103	-0.159
<i>v</i>	42.618	42.617	42.618	42.617	42.618	42.609	41.949
<i>working capital</i>	76.660	76.658	76.659	76.658	76.659	76.644	75.455
Standard deviation (in percent)							
<i>gdp</i>	3.91	3.94	3.94	3.94	3.94	3.94	3.85
<i>c</i>	3.95	4.03	4.02	4.03	4.02	4.03	3.69
<i>i</i>	13.33	13.33	13.33	13.33	13.33	13.32	13.45
<i>nx/gdp</i>	2.90	2.94	2.94	2.94	2.94	2.94	2.58
<i>k</i>	4.40	4.49	4.49	4.49	4.49	4.50	4.31
<i>b/gdp</i>	18.72	19.62	19.47	19.62	19.47	19.45	8.90
<i>q</i>	3.20	3.20	3.20	3.20	3.20	3.20	3.23
<i>leverage ratio</i>	8.79	9.22	9.15	9.22	9.15	9.14	4.07
<i>v</i>	5.87	5.89	5.89	5.89	5.89	5.89	5.84
<i>working capital</i>	4.33	4.35	4.35	4.35	4.35	4.36	4.26
Correlation with GDP							
<i>gdp</i>	1.000	1.000	1.000	1.000	1.000	1.000	1.000
<i>c</i>	0.849	0.842	0.844	0.842	0.844	0.844	0.931
<i>i</i>	0.646	0.641	0.641	0.641	0.641	0.641	0.641
<i>nx/gdp</i>	-0.122	-0.117	-0.118	-0.117	-0.118	-0.120	-0.184
<i>k</i>	0.757	0.761	0.761	0.761	0.761	0.761	0.744
<i>b/gdp</i>	-0.133	-0.120	-0.119	-0.120	-0.119	-0.117	-0.298
<i>q</i>	0.400	0.387	0.387	0.387	0.387	0.387	0.406
<i>leverage ratio</i>	-0.125	-0.111	-0.111	-0.111	-0.111	-0.108	0.258
<i>v</i>	0.831	0.832	0.832	0.832	0.832	0.832	0.823
<i>working capital</i>	0.994	0.994	0.994	0.994	0.994	0.994	0.987
First-order autocorrelation							
<i>gdp</i>	0.823	0.825	0.824	0.825	0.824	0.825	0.815
<i>c</i>	0.823	0.830	0.829	0.830	0.829	0.829	0.766
<i>i</i>	0.500	0.501	0.500	0.501	0.500	0.500	0.483
<i>nx/gdp</i>	0.589	0.601	0.598	0.601	0.598	0.598	0.447
<i>k</i>	0.964	0.962	0.962	0.962	0.962	0.962	0.963
<i>b/gdp</i>	0.989	0.990	0.990	0.990	0.990	0.990	0.087
<i>q</i>	0.444	0.447	0.446	0.447	0.446	0.446	0.428
<i>leverage ratio</i>	0.991	0.992	0.992	0.992	0.992	0.992	0.040
<i>v</i>	0.776	0.777	0.777	0.777	0.777	0.777	0.764
<i>working capital</i>	0.800	0.801	0.801	0.801	0.801	0.801	0.777
Prob. of Sudden Stops	1.98%	1.99%	2.03%	1.99%	2.04%	2.05%	3.32%
<i>(b) Performance metrics</i>							
Bonds Euler Equation							
<i>Max Log10 Abs. Euler Error</i>	-3.58	-3.58	-3.58	-3.58	-3.58	-3.52	-
<i>At Grid Points (b, k, s)</i>	(1, 11, 3)	(1, 6, 3)	(1, 6, 3)	(1, 6, 3)	(1, 6, 3)	(2, 1, 7)	-
<i>Mean Log10 Abs. Euler Error</i>	-14.45	-14.45	-12.41	-14.27	-12.39	-12.35	-
Capital Euler Equation							
<i>Max Log10 Abs. Euler Error</i>	-15.38	-15.37	-15.42	-15.37	-15.42	-4.06	-
<i>At Grid Points (b, k, s)</i>	(72, 1, 7)	(72, 1, 7)	(32, 1, 7)	(72, 1, 7)	(32, 1, 7)	(1, 1, 7)	-
<i>Mean Log10 Abs. Euler Error</i>	-16.22	-16.22	-16.07	-16.04	-16.21	-12.45	-
Grid size (#b, #k)	(72, 60)	(72, 30)	(72, 30)	(72, 30)	(72, 30)	(72, 30)	(80, 60)
Seconds elapsed	1985	810	3735	956	4136	1986	-
Relative to FiPIt	2.5	1.0	4.6	1.2	5.1	2.5	-
Number of iterations	196	196	190	178	207	94	-

Notes: Column (1) and Column (2) are for the *FiPIt* algorithm, fixed-point iteration is used for both the bonds decision rule and the price of capital. Column (3) is for the *TIFS* method, which uses the time iteration method for the bonds decision rule and the forward solution of the capital Euler equation for the price of capital. Column (4) is for the *FPIFS* method, which uses fixed-point iteration for the bonds decision rule and the forward solution of the capital Euler equation for the price of capital. Column (5) is for the *TIFPI* method, which uses time iteration for the decision rule for bonds and fixed-point iteration for the price of capital. Column (6) is for the *FTI* method, which solves the bonds decision rule and the price of capital by solving the Euler equations for bonds and capital as two simultaneous non-linear equations. Sudden Stop states are defined as in Mendoza (2010): states (b, k, s) such that $\mu(b, k, s) > 0$ and the trade balance-GDP ratio is at least 2 percentage points above its value in the RBC model.

equilibrium functions produced by each solution method relative to the *FiPIt* solution. Table 3.3 shows the maximum and mean of the absolute value of the point-wise differences of the functions as a ratio of the corresponding *FiPIt* solution. The differences are generally negligible, except for the maximum differences for b' and i in the *FTI* solution, which reach 9.94 and 2.25 respectively in states in which the corresponding denominator is very close to zero. Still, as shown in Table 3.2 this makes little difference in first moments and is nearly irrelevant for second- and higher-order moments.

Table 3.3: Absolute Values of Differences in Equilibrium Functions Relative to *FiPIt* Solution

Differences Relative to <i>FiPIt</i> Method				
	(1) TIFS	(2) FPIFS	(3) TIFPI	(4) FTI
Max Difference				
b'	3.17e+00	5.71e-02	3.21e+00	9.94e+00
k'	2.23e-04	7.17e-07	2.23e-04	1.10e-02
q	6.04e-04	1.99e-06	6.04e-04	1.30e-01
c	9.72e-05	7.08e-07	9.72e-05	1.09e-02
i	8.75e-02	5.79e-05	8.74e-02	2.25e+00
L	4.92e-05	3.94e-07	4.92e-05	2.57e-03
v	9.08e-05	7.27e-07	9.08e-05	4.76e-03
gdp	3.27e-05	2.61e-07	3.27e-05	1.77e-03
Mean Difference				
b'	6.29e-04	1.10e-05	6.37e-04	4.35e-03
k'	1.13e-05	2.76e-07	1.15e-05	9.81e-05
q	3.16e-05	7.61e-07	3.22e-05	5.64e-04
c	2.00e-05	3.22e-07	2.03e-05	7.10e-05
i	1.74e-04	3.27e-06	1.76e-04	1.51e-03
L	1.22e-06	3.27e-09	1.22e-06	2.38e-05
v	2.24e-06	6.03e-09	2.25e-06	4.40e-05
gdp	8.16e-07	2.19e-09	8.18e-07	1.62e-05

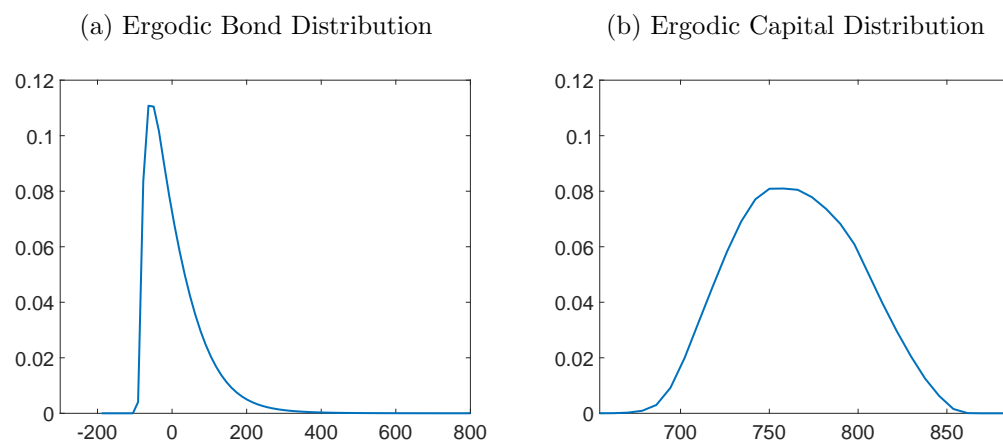
Figure 3.1 shows the ergodic marginal distributions of bonds and capital, and the ergodic joint marginal distribution of both variables produced by the *FiPIt* solution.

These plots are generated using the full ergodic distribution of (b, k, s) , which *FiPit* computes using a procedure that iterates to convergence on the law of motion of the conditional distribution of (b, k, s) (starting from an arbitrary initial condition) taking into account the fact the the decision rules of capital and bonds are generally off the nodes of the corresponding grids. Full details are provided in the Appendix. The long-run moments listed in Table 3.2 were produced using this distribution. The distributions produced by all the other solution methods are visually identical, and hence we only show the ones for the *FiPit* case. Relative to the distributions that the RBC model would produce, the distribution of bonds shifts to the right because of the credit constraint and the stronger precautionary saving incentives. The distribution of capital shows higher dispersion and a fatter left tail because of the fire-sales of capital in states in which the constraint binds.

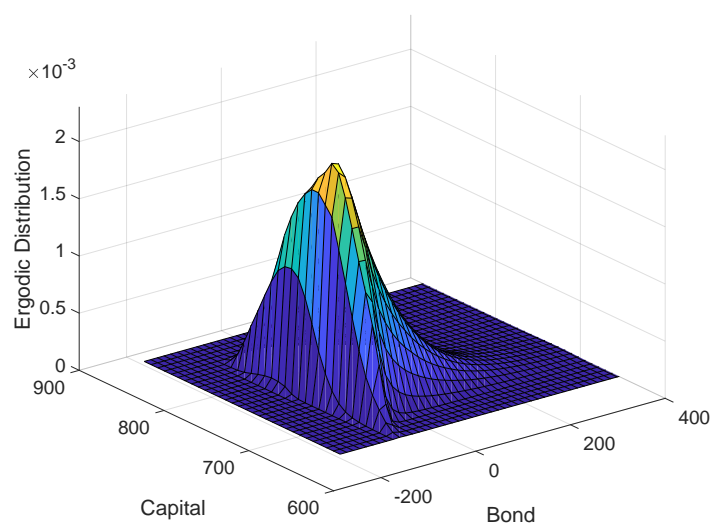
We also examined the recursive equilibrium functions to evaluate the relevance of the global solution to capture non-linearities. Figure 3.2 shows the decision rules of bonds and capital, the pricing function of capital and the multiplier of the credit constraint across the full state space of endogenous states, $\mathbf{B} \otimes \mathbf{K}$, with s evaluated for a state with low TFP, high interest rate, and high input prices. We show results for the Sudden Stops model and for the RBC variant, and provide only the *FiPit* results because the other methods yield visually identical graphs. The equilibrium functions of the Sudden Stops model show significant non-linearities, whereas the RBC outcomes are approximately linear. The non-linearities result from the fire-sales of capital when the constraint binds, the resulting collapse in the price of capital, and the associated sharp reversal in the bond position as borrowing capacity collapses.

The sharp curvature of these non-linear solutions highlights the advantages of using a finite-state-space solution method, instead of a collocation method, as well as the importance of solving using first-order conditions and approximately-continuous

Figure 3.1: Long-run Distributions of the Sudden Stops Model Solved with FiPIt



(c) Ergodic Distribution of Bonds and Capital



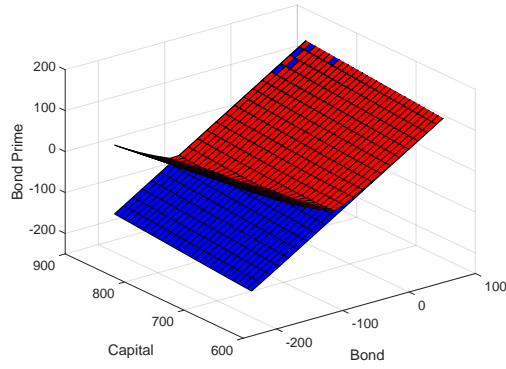
decision rules. Decision rules that capture accurately the non-linearities implied by occasionally binding constraints are critical for quantifying the positive and normative implications of this class of models, including Sudden Stops models. For their positive implications, the magnitude, dynamics and frequency of financial crises depends critically on the behavior of decision rules near and at the constraint. For the normative implications, quantifying the size of distortions induced by the credit constraint and the properties of optimal policies to tackle them hinges critically on how likely and how severely is the credit constraint expected to bind at $t+1$ in a state in which it does not bind at t (see Bianchi and Mendoza, 2018).

The plots of equilibrium functions do not control for whether particular (b, k, s) triples have positive probability in the stochastic steady state. States with zero-probability are irrelevant in the long run, and if this is the case in the region where equilibrium functions are non-linear, the non-linearities would be of less relevance than what the equilibrium functions suggest. To assess this issue, we follow Mendoza (2010) to calculate impact amplification coefficients and report the results in Table 3.4. These coefficients measure the excess response of macro variables across the Sudden Stops and RBC solutions for each triple (b, k, s) , separating the state space into Sudden Stop (SS) and non-Sudden Stop (NSS) regions.¹¹ The averages shown in the SS and NSS columns of the table are computed using the limiting distribution of (b, k, s) of the Sudden Stops model. The results in the SS column measure amplification on impact when a crisis occurs. Differences across the SS and NSS columns illustrate asymmetry, namely the amount by which shocks of identical magnitudes generate

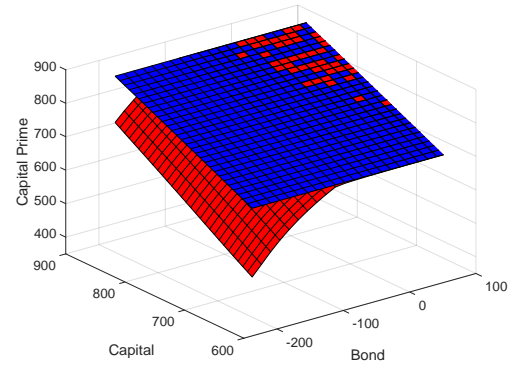
¹¹A triple (b, k, s) belongs in the SS set if the trade balance-GDP ratio in the Sudden Stops model is 2 percentage points or more above its value in the RBC model, otherwise it belongs in the NSS region. The amplification coefficients for each variable at a given (b, k, s) are calculated as differences relative to their values in the RBC model in the same state and expressed in percent of the unconditional mean of the variable also in the RBC model. For variables defined in ratios, the coefficient is the difference in the Sudden Stops model relative to the RBC model.

Figure 3.2: Equilibrium Recursive Functions of the Sudden Stops & RBC Models

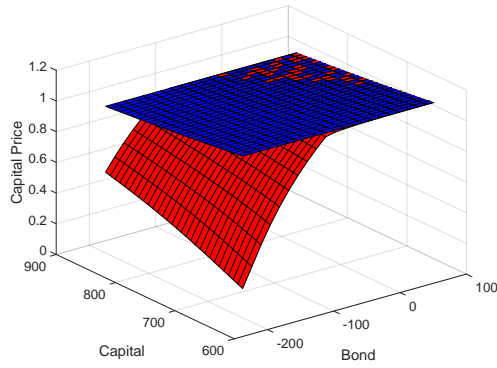
(a) Bonds decision rule



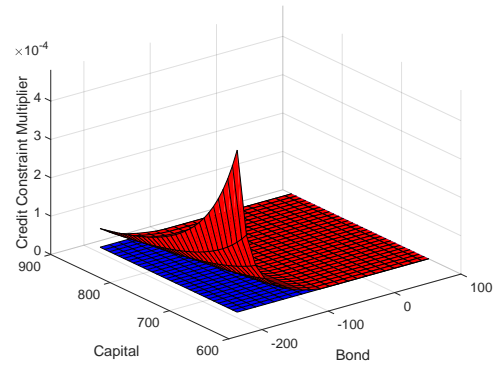
(b) Capital decision rule



(c) Price of capital



(d) Credit Constraint Multiplier



Notes: All plots show solutions obtained with the *FiPIt* method. Surface plots in red (blue) are for the SS (RBC) model.

different effects when the collateral constraint is present and active v. when is not.

Table 3.4 compares amplification coefficients produced by the *FiPIt* and *FTI* solutions (the other methods yield nearly identical results). The coefficients differ very marginally and in most instances they are the same up to the second decimal. The table shows that the Sudden Stops model yields significant amplification and asymmetry. Amplification coefficients on factor allocations and output are relatively smaller, because on impact at date- t when the credit constraint binds it can only affect them via its effect on working capital financing and hence on labor and imported inputs. In turn, this is due to the absence of the wealth effect on labor supply implied by the utility function specification and to the fact that the date- t capital stock is pre-determined.

The *FiPIt* method yields more accurate results than those produced by the solution method used in Mendoza (2010). The results in Table 3.4 are qualitatively similar to those reported in Table 4 of Mendoza’s paper, but quantitatively there are significant differences. Differences in model structure (i.e. endogenous v. exogenous discounting) play some role, but the bulk of the differences is due to differences in the solution methods. Mendoza solved for decision rules forced to be on grid nodes using value function iteration, while *FiPIt* solves for interpolated decision rules and iterates on the model’s optimality conditions. *FiPIt* yields coefficients for “supply side” variables (i.e. GDP, labor, imported inputs and working capital) that are smaller, while those for the rest of the variables (particularly investment and the price of capital) are larger. Moreover, for supply-side variables in the NSS region *FiPIt* yields near-zero amplification while Mendoza reports figures in the -0.29 to -0.11 range. The *FiPIt* results are the correct ones because the amplification coefficients for these variables should indeed differ from zero only due to numerical approximation error. Since k is pre-determined at each date t and there is no wealth effect on labor supply,

when $\mu(b, k, s) = 0$ the set of optimality conditions is the same in the RBC and Sudden Stops models and in both cases all supply-side variables depend only on (k, s) . The coefficients around -0.11 to -0.29 that Mendoza obtained result from non-trivial numerical approximation errors due to inaccuracies of the solution algorithm when averaging outcomes for states in which the NSS and SS regions are adjacent and in determining the value of $\mu(b, k, s)$ when assigning (b, k, s) triples to the SS and NSS sets.

Table 3.4: Amplification and Asymmetry of Sudden Stop events

	(1)		(2)	
	FiPIt		FTI	
	SS	NSS	SS	NSS
<i>gdp</i>	-0.777	-0.001	-0.789	-0.001
<i>c</i>	-3.849	-0.255	-3.882	-0.260
<i>i</i>	-24.965	-1.036	-25.384	-1.089
<i>q</i>	-6.090	-0.253	-6.194	-0.266
<i>nx/gdp</i>	4.033	0.233	4.047	0.238
<i>b'/gdp</i>	4.215	0.251	4.229	0.257
<i>k'/gdp</i>	-1.667	-0.105	-1.680	-0.110
<i>lev. ratio</i>	1.166	0.081	1.167	0.082
<i>L</i>	-1.178	-0.001	-1.196	-0.002
<i>v</i>	-2.146	-0.003	-2.180	-0.003
<i>w. cap</i>	-2.160	-0.003	-2.193	-0.003

Notes: Sudden Stop (SS) states are defined as states in which the collateral constraint binds and the trade balance-GDP ratio in the Sudden Stop model is more than 2 percentage points above the trade balance-GDP ratio of the RBC model. The coefficients are computed as mean differences relative to the RBC model in percent of the RBC unconditional averages.

3.4.2 Robustness Analysis & Credit Constraint Variations

The last set of experiments evaluates the robustness and stability of the *FiPIt* algorithm by examining its performance relative to the time iteration method for various

parameter changes. This is important in light of the potential instability of fixed-point iteration methods. As documented below, the *FiPIt* method remains stable and continues to outperform the *FTI* method in all the experiments. We also provide results for the RBC variant of the model and for variations of the credit constraint for which *FiPIt* does not require using a non-linear solver in states in which the constraint binds and found even larger gains in execution time in both instances.

Tables 3.5, 3.6 and 3.7 show long-run moments and performance metrics obtained by solving the model using the *FiPIt* and *FTI* methods for these parameter changes: (a) removing working capital ($\phi = 0$); (b) lowering the discount factor ($\beta = 0.91$); (c) reducing the collateral coefficient ($\kappa = 0.15$); (d) increasing the collateral coefficient ($\kappa = 0.25$); (e) setting the collateral coefficient so that the constraint never binds ($\kappa \geq 1.0$), which yields the RBC solution; (f) increasing the labor disutility coefficient ($\omega = 2.5$); and (g) increasing the relative risk aversion coefficient ($\sigma = 3.0$). For each parameter variation, the grids of capital and bonds were re-sized to obtain the fastest solution that does not distort the quantitative results, using identical grids for the *FiPIt* and *FTI* solutions. Still, this resulted in grids of about the same dimensions as before: 71 or 72 nodes in **B** and 30 nodes in **K**, except for case (e) with the RBC model, for which 80 nodes in **B** were needed, and case (f) that needed only 62 nodes in **B**.¹²

The dominance of the *FiPIt* method is robust to all these parameter changes, and in all cases the algorithm is stable and yields solutions nearly identical to the *FTI* results. Comparing across the cases in which the root-finder is needed to solve allocations when the credit constraint binds (i.e. excluding case (a)), *FTI* is 2.0 to

¹²When solving the RBC variant of the model, the bonds grid is extended to accommodate larger debt positions that are part of the equilibrium solution. In this case, **B** consists of 80 nodes spanning the $[-300.0, 800]$ interval. The upper bound is the same as before, but the lower bound of -300 is significantly smaller (relative to -188.6 used in the solutions reported earlier).

6.0 times slower than *FiPIt* depending on which scenario is considered. Comparing v. the scenario in which *FiPIt* does not need the root-finder when the constraint binds (Col. (2) of Table 3.5), *FTI* is 13.0 times slower, and for solving the RBC model, which also does not need a root-finder (Col. (6) of Table 3.6), *FTI* is 18.1 times slower. In both of these instances, *FiPIt* solves in about 2 minutes. Moreover, in most cases *FiPIt* did not require changing the values of the dampening parameters for the updates of the decision rule for bonds ($\rho^b = 1$), the credit constraint multiplier ($\rho^\mu = 1$) and the pricing function ($\rho^q = 0.3$).

It is worth noting that the time iteration solutions required about the same number of iterations (between 87 and 100) and execution time in all the experiments except case (f), which has the smaller **B** grid and used about the same number of iterations but solved faster than the other time iteration solutions. There is more variation in both number of iterations and execution times in the *FiPIt* solutions, but the two tend to move together: The slowest solution was for case (b) which took 1,130 seconds and 244 iterations.

For the case without working capital (case (a)), Column (2) shows the results that *FiPIt* yields when the code is modified to take into account that a root-finder is not needed to solve when the credit constraint binds, as explained in Section 3.3 (since the constraint is now of the form $b'_{j+1}(b, k, s)/R \geq -\kappa \hat{q}_j(b, k, s)k'_j(b, k, s)$). We also solved an additional experiment with an alternative credit constraint in the same class that does not require a non-linear solver: $b'_{j+1}(b, k, s)/R \geq \varphi$ with φ set one standard deviation below the average of b' in the limiting distribution of the RBC model. These experiments illustrate the large additional gain in speed that *FiPIt* yields when used to solve models with constraints like these. In Case (a), the *FiPIt* solution is obtained in almost one-third of the time taken by the *FiPIt* algorithm that uses the non-linear solver, which implies that *FiPIt* is faster than the time iteration

solution by a factor of 13.0 (v. 5.6 with the *FiPIt* algorithm that uses the non-linear solver). In the case with the constraint given by φ , the *FiPIt* solution is faster than the time iteration method by a factor of 17.9.

3.5 Conclusions

FiPIt is a simple and fast algorithm designed to solve macroeconomic models with two endogenous state variables and occasionally binding constraints using widely used software. The algorithm applies fixed-point iteration on the Euler equations and by doing so it avoids solving the Euler equations as a non-linear system, as with the standard time iteration method, and does not require interpolation of decision rules over irregular grids, as with the endogenous grids method. Analytic solutions are obtained for recursive equilibrium functions in each iteration of the algorithm, and standard bi-linear interpolation for obtaining these analytic solutions remains applicable.

The *FiPIt* algorithm can handle a large class of occasionally binding constraints, including constraints set to fixed values as well as constraints that depend on endogenous variables. If the constraints are such that equilibrium allocations and prices when the constraints bind must be solved jointly with their associated multipliers, *FiPIt* does need a root-finder in states in which the constraint bind, but for a large class of constraints the two can be solved separately and *FiPIt* does not require a non-linear solver anywhere. In contrast, the endogenous grid method requires a root finder whenever the constraint binds.

We documented the performance gains and accuracy of *FiPIt* by comparing the solutions it produces for a Sudden Stops model of a small open economy vis-a-vis solutions obtained with the time iteration method, and hybrid methods that combine

Table 3.5: Sudden Stops Model Variations: Working Capital & Discounting

	(a) Working Capital $\phi = 0$			(b) Discount factor $\beta = 0.91$	
	(1) FiPIt	(2) FiPIt (no root-finder when $\mu > 0$)	(3) FTI	(4) FiPIt	(5) FTI
<i>(a) Long-run moments</i>					
Mean					
<i>gdp</i>	406.361	406.361	406.291	368.772	368.159
<i>c</i>	282.681	282.681	282.564	255.219	254.877
<i>i</i>	69.847	69.847	69.824	59.863	59.678
<i>nx/gdp</i>	0.015	0.015	0.015	0.029	0.029
<i>k</i>	792.035	792.035	791.780	678.870	676.802
<i>b/gdp</i>	-0.202	-0.202	-0.205	-0.161	-0.160
<i>q</i>	1.000	1.000	1.000	1.000	1.000
<i>leverage ratio</i>	-0.095	-0.095	-0.097	-0.195	-0.195
<i>v</i>	45.079	45.079	45.072	39.798	39.729
<i>working capital</i>	0.000	0.000	0.000	71.585	71.460
Standard deviation (in percent)					
<i>gdp</i>	3.71	3.71	3.71	3.93	3.93
<i>c</i>	3.82	3.82	3.82	3.91	3.92
<i>i</i>	13.16	13.16	13.16	12.17	12.16
<i>nx/gdp</i>	2.94	2.94	2.93	2.12	2.11
<i>k</i>	4.44	4.44	4.45	4.53	4.54
<i>b/gdp</i>	20.06	20.06	19.88	2.28	2.26
<i>q</i>	3.16	3.16	3.16	2.88	2.88
<i>leverage ratio</i>	9.47	9.47	9.39	0.69	0.68
<i>v</i>	5.42	5.42	5.43	5.97	5.97
<i>working capital</i>	-	-	-	4.38	4.39
Correlation with GDP					
<i>gdp</i>	1.000	1.000	1.000	1.000	1.000
<i>c</i>	0.820	0.820	0.823	0.969	0.969
<i>i</i>	0.593	0.593	0.594	0.713	0.713
<i>nx/gdp</i>	-0.083	-0.083	-0.085	-0.310	-0.311
<i>k</i>	0.775	0.775	0.776	0.754	0.754
<i>b/gdp</i>	-0.070	-0.070	-0.067	-0.093	-0.096
<i>q</i>	0.334	0.334	0.334	0.442	0.441
<i>leverage ratio</i>	-0.076	-0.076	-0.073	-0.024	-0.030
<i>v</i>	0.795	0.795	0.795	0.833	0.833
<i>working capital</i>	-	-	-	0.988	0.988
First-order autocorrelation					
<i>gdp</i>	0.834	0.834	0.834	0.818	0.818
<i>c</i>	0.860	0.860	0.860	0.759	0.759
<i>i</i>	0.501	0.501	0.500	0.330	0.330
<i>nx/gdp</i>	0.608	0.608	0.606	0.068	0.068
<i>k</i>	0.962	0.962	0.962	0.970	0.970
<i>b/gdp</i>	0.991	0.991	0.991	0.423	0.415
<i>q</i>	0.446	0.446	0.446	0.234	0.235
<i>leverage ratio</i>	0.992	0.992	0.992	0.686	0.679
<i>v</i>	0.788	0.788	0.788	0.756	0.757
<i>working capital</i>	-	-	-	0.755	0.756
P(S.S)	1.43%	1.43%	1.49%	39.98%	40.44%
<i>(b) Performance metrics</i>					
Bonds Euler Equation					
<i>Max Log10 Abs. Euler Error</i>	-4.17	-4.17	-4.13	-3.56	-3.51
<i>At Grid Points (b, k, s)</i>	(1, 1, 3)	(1, 1, 3)	(1, 1, 3)	(3, 1, 3)	(3, 1, 3)
<i>Mean Log10 Abs. Euler Error</i>	-15.53	-15.53	-13.25	-18.96	-12.07
Capital Euler Equation					
<i>Max Log10 Abs. Euler Error</i>	15.43	15.43	-6.36	15.40	-4.18
<i>At Grid Points (b, k, s)</i>	(72, 1, 7)	(72, 1, 7)	(1, 1, 7)	(72, 1, 7)	(2, 1, 7)
<i>Mean Log10 Abs. Euler Error</i>	-16.17	-16.17	-12.77	-22.43	-12.23
Grid size (#b, #k)	(72, 30)	(72, 30)	(72, 30)	(72, 30)	(72, 30)
Seconds elapsed	297	123	1593	1220	2405
Relative to FiPIt	2.4	1.0	13.0	1.0	2.0
Number of iterations	200	200	95	244	87

Notes: Columns (1) are *FiPIt* solutions and Columns (3) are time iteration (*FTI*) solutions. Column (2) shows results for the model with $\phi = 0$ obtained with the *FiPIt* algorithm without using a non-linear solver when $\mu > 0$, since it is not needed.

Table 3.6: Sudden Stops Model Variations: Collateral Coefficient

	(c) Lower Coll. Coeff. $\kappa = 0.15$		(d) Higher Coll. Coeff. $\kappa = 0.25$		(e) RBC Non-Binding Coll. Coeff. $\kappa \geq 1$	
	(1) FiPIt	(2) FTI	(3) FiPIt	(4) FTI	(5) FiPIt	(6) FTI
<i>(a) Long-run moments</i>						
Mean						
<i>gdp</i>	393.503	393.433	393.728	393.659	393.847	393.813
<i>c</i>	276.545	276.436	271.926	271.807	264.021	263.871
<i>i</i>	67.442	67.420	67.517	67.495	67.530	67.518
<i>nx/gdp</i>	0.007	0.008	0.022	0.022	0.045	0.046
<i>k</i>	764.752	764.503	765.564	765.316	765.885	765.759
<i>b/gdp</i>	0.109	0.106	-0.070	-0.073	-0.372	-0.377
<i>q</i>	1.000	1.000	1.000	1.000	1.000	1.000
<i>leverage ratio</i>	-0.057	-0.059	-0.142	-0.144	-0.286	-0.288
<i>v</i>	42.604	42.596	42.630	42.622	42.649	42.646
<i>working capital</i>	76.634	76.620	76.681	76.667	76.716	76.710
Standard deviation (in percent)						
<i>gdp</i>	3.91	3.91	3.96	3.96	3.99	3.99
<i>c</i>	3.87	3.87	4.20	4.20	5.15	5.14
<i>i</i>	13.23	13.22	13.43	13.43	13.51	13.51
<i>nx/gdp</i>	2.90	2.89	3.02	3.01	3.53	3.52
<i>k</i>	4.43	4.44	4.54	4.55	4.65	4.65
<i>b/gdp</i>	19.10	18.92	20.73	20.55	30.28	30.04
<i>q</i>	3.18	3.18	3.23	3.23	3.24	3.23
<i>leverage ratio</i>	8.96	8.88	9.76	9.68	14.42	14.31
<i>v</i>	5.87	5.87	5.91	5.91	5.91	5.91
<i>working capital</i>	4.33	4.33	4.38	4.38	4.40	4.40
Correlation with GDP						
<i>gdp</i>	1.000	1.000	1.000	1.000	1.000	1.000
<i>c</i>	0.842	0.844	0.836	0.839	0.773	0.776
<i>i</i>	0.641	0.642	0.640	0.640	0.640	0.640
<i>nx/gdp</i>	-0.085	-0.088	-0.145	-0.148	-0.227	-0.230
<i>k</i>	0.758	0.758	0.763	0.763	0.767	0.767
<i>b/gdp</i>	-0.169	-0.167	-0.072	-0.069	0.090	0.093
<i>q</i>	0.390	0.390	0.385	0.385	0.381	0.381
<i>leverage ratio</i>	-0.165	-0.164	-0.060	-0.057	0.112	0.115
<i>v</i>	0.831	0.831	0.833	0.833	0.834	0.834
<i>working capital</i>	0.995	0.995	0.994	0.994	0.995	0.995
First-order autocorrelation						
<i>gdp</i>	0.823	0.823	0.826	0.826	0.830	0.830
<i>c</i>	0.823	0.822	0.837	0.837	0.885	0.885
<i>i</i>	0.500	0.499	0.501	0.501	0.516	0.516
<i>nx/gdp</i>	0.602	0.600	0.607	0.604	0.711	0.709
<i>k</i>	0.961	0.961	0.962	0.962	0.964	0.964
<i>b/gdp</i>	0.989	0.988	0.991	0.991	0.997	0.997
<i>q</i>	0.447	0.447	0.446	0.446	0.460	0.460
<i>leverage ratio</i>	0.990	0.990	0.993	0.993	0.997	0.997
<i>v</i>	0.776	0.776	0.777	0.777	0.780	0.780
<i>working capital</i>	0.800	0.800	0.802	0.802	0.807	0.808
P(S.S)	2.90%	2.98%	1.34%	1.45%	-	-
<i>(b) Performance metrics</i>						
<i>Max Log10 Abs. Euler Error</i>	-3.59	-3.53	-3.57	-3.52	-6.93	-6.93
<i>At Grid Points (b, k, s)</i>	(1, 1, 3)	(1, 1, 3)	(1, 2, 3)	(1, 1, 3)	(1, 1, 3)	(1, 1, 3)
<i>Mean Log10 Abs. Euler Error</i>	-14.11	-12.09	-14.96	-12.71	-15.96	-13.33
Capital Euler Equation						
<i>Max Log10 Abs. Euler Error</i>	15.37	-4.22	15.37	-4.14	15.42	-9.91
<i>At Grid Points (b, k, s)</i>	(71, 1, 7)	(1, 1, 7)	(72, 1, 7)	(1, 1, 7)	(2, 1, 7)	(2, 1, 3)
<i>Mean Log10 Abs. Euler Error</i>	-16.28	-12.38	-16.24	-12.56	-16.36	-12.74
Grid size (#b, #k)	(71, 30)	(71, 30)	(72, 30)	(72, 30)	(80, 30)	(80, 30)
Seconds elapsed	1066	2668	657	1604	100	1808
Relative to FiPIt	1.0	2.5	1.0	2.4	1.0	18.1
Number of iterations	196	94	196	94	201	94

Notes: Columns (1) are for the *FiPIt* algorithm. Columns (2) are for the full time iteration method (*FTI*).

Table 3.7: Sudden Stops Model Variations: Labor Elasticity & Risk Aversion

	(f) Higer Labor Coeff. $\omega = 2.5$		(g) Higher Risk Aversion $\sigma = 3$	
	(1) FiPIt	(2) FTI	(3) FiPIt	(4) FTI
<i>(a) Long-run moments</i>				
Mean				
<i>gdp</i>	110.477	110.441	393.797	393.722
<i>c</i>	78.995	78.897	277.504	277.357
<i>i</i>	18.937	18.924	67.534	67.509
<i>nx/gdp</i>	-0.007	-0.006	0.005	0.005
<i>k</i>	214.735	214.586	765.801	765.531
<i>b/gdp</i>	0.289	0.278	0.138	0.134
<i>q</i>	1.000	1.000	1.000	1.000
<i>leverage ratio</i>	0.027	0.023	-0.044	-0.045
<i>v</i>	11.960	11.956	42.639	42.630
<i>working capital</i>	21.514	21.507	76.697	76.682
Standard deviation (in percent)				
<i>gdp</i>	3.33	3.33	3.97	3.98
<i>c</i>	6.31	6.24	4.35	4.35
<i>i</i>	13.11	13.11	13.46	13.46
<i>nx/gdp</i>	5.39	5.31	3.25	3.24
<i>k</i>	4.32	4.34	4.59	4.61
<i>b/gdp</i>	56.17	55.03	28.69	28.55
<i>q</i>	3.17	3.17	3.23	3.23
<i>leverage ratio</i>	26.48	25.94	13.55	13.49
<i>v</i>	5.40	5.41	5.91	5.92
<i>working capital</i>	3.76	3.77	4.39	4.40
Correlation with GDP				
<i>gdp</i>	1.000	1.000	1.000	1.000
<i>c</i>	0.451	0.463	0.744	0.747
<i>i</i>	0.614	0.614	0.638	0.639
<i>nx/gdp</i>	-0.077	-0.084	-0.059	-0.062
<i>k</i>	0.755	0.756	0.765	0.765
<i>b/gdp</i>	-0.039	-0.034	-0.058	-0.054
<i>q</i>	0.366	0.365	0.381	0.381
<i>leverage ratio</i>	-0.043	-0.037	-0.048	-0.044
<i>v</i>	0.797	0.797	0.834	0.834
<i>working capital</i>	0.992	0.992	0.995	0.995
First-order autocorrelation				
<i>gdp</i>	0.820	0.821	0.828	0.828
<i>c</i>	0.941	0.939	0.894	0.894
<i>i</i>	0.489	0.489	0.506	0.506
<i>nx/gdp</i>	0.857	0.852	0.733	0.731
<i>k</i>	0.960	0.960	0.963	0.963
<i>b/gdp</i>	0.995	0.995	0.995	0.995
<i>q</i>	0.435	0.435	0.450	0.450
<i>leverage ratio</i>	0.995	0.995	0.996	0.996
<i>v</i>	0.768	0.769	0.779	0.779
<i>working capital</i>	0.793	0.794	0.804	0.805
P(S.S)	3.38%	3.66%	1.29%	1.35%
<i>(b) Performance metrics</i>				
Bonds Euler Equation				
<i>Max Log10 Abs. Euler Error</i>	-3.43	-3.37	-3.82	-3.75
<i>At Grid Points (b, k, s)</i>	(1, 1, 7)	(1, 1, 7)	(2, 1, 7)	(2, 1, 7)
<i>Mean Log10 Abs. Euler Error</i>	-14.27	-13.16	-15.11	-12.41
Capital Euler Equation				
<i>Max Log10 Abs. Euler Error</i>	14.09	-4.46	16.30	-3.90
<i>At Grid Points (b, k, s)</i>	(62, 1, 7)	(1, 1, 7)	(71, 1, 7)	(1, 1, 7)
<i>Mean Log10 Abs. Euler Error</i>	-14.76	-12.52	-17.13	-12.56
Grid size (#b, #k)	(62, 30)	(62, 30)	(71, 30)	(71, 30)
Seconds elapsed	283	1700	1108	3533
Relative to FiPIt	1.0	6.0	1.0	3.2
Number of iterations	163	98	246	100

Notes: Columns (1) are for the *FiPIt* algorithm. Columns (2) are for the full time iteration method (*FTI*).

fixed-point and time iteration techniques. In addition, we explored the robustness of our algorithm by documenting solutions for seven parameter variations, including an RBC model in which the constraint never binds. The algorithm was coded in Matlab and executed in a standard Windows laptop. In all cases, *FiPIt* produced results nearly identical to time iteration results with large gains in speed and comparable accuracy as measured by Euler equation errors. Time iteration solutions exceeded the execution time of the *FiPIt* solutions by factors of 2.0 to 18.1. The largest gains were obtained in cases in which *FiPIt* does not use root-finders anywhere, which include the RBC solution and a variation of the Sudden Stops model without working capital. In these cases, solving for allocations when the constraint binds does not require a non-linear solver. Time iteration took 18.1 and 13 times longer than *FiPIt* to solve the RBC model and the Sudden Stops model without working capital, respectively. For the baseline Sudden Stops model, which does need the solver to determine allocations when the constraint binds, time iteration took 2.5 times longer than *FiPIt*.

The *FiPIt* algorithm can be extended to other models with two endogenous states, since applying it requires mainly a fixed-point strategy to iterate on recursive functions using Euler equations. In this paper, *FiPIt* was applied to the Euler equation for bonds to solve for the bonds decision rule and to the Euler equation for capital to solve for the price of capital. The Tobin's Q investment optimality condition was then used to determine the decision rule for capital. It is possible to re-arrange the solution in other ways that *FiPIt* may still accommodate, for example conjecturing the bonds and capital decision rules and using the two Euler equations to solve for their updates. Applying these principles to other models with two endogenous state variables so that they can be solved using *FiPIt* seems relatively straightforward. We provide a brief sketch of four examples in the online Appendix.

Performance gains using *FiPIt* are likely to be even larger if the algorithm is

coded in languages that are more efficient than Matlab at handling high-dimensional, sequential loops and parallel optimization, such as Julia, Fortran or Python. The large gains in speed and simplicity of the algorithm also open up the possibility of exploring research topics such as Bayesian estimation of models of financial crisis driven by occasionally binding collateral constraints.

References

- Aruoba, S Borağan et al. (2021). “Piecewise-linear approximations and filtering for DSGE models with occasionally-binding constraints”. In: *Review of Economic Dynamics*.
- Bianchi, Javier, Chenxin Liu, and Enrique G Mendoza (2016). “Fundamentals news, global liquidity and macroprudential policy”. In: *Journal of International Economics* 99, S2–S15.
- Bianchi, Javier and Enrique G Mendoza (2018). “Optimal Time-Consistent Macroprudential Policy”. In: *Journal of Political Economy* 126.2, pp. 588–634.
- Boz, Emine and Enrique G Mendoza (2014). “Financial innovation, the discovery of risk, and the US credit crisis”. In: *Journal of Monetary Economics* 62, pp. 1–22.
- Brumm, Johannes and Michael Grill (2014). “Computing equilibria in dynamic models with occasionally binding constraints”. In: *Journal of Economic Dynamics and Control* 38, pp. 142–160.
- Carroll, Christopher D (2006). “The method of endogenous gridpoints for solving dynamic stochastic optimization problems”. In: *Economics letters* 91.3, pp. 312–320.
- (2011). “Lecture Notes On Solution Methods for Representative Agent Dynamic Stochastic Optimization Problems”. In: *Mimeo*.

- Coleman, Wilbur John (1990). “Solving the stochastic growth model by policy-function iteration”. In: *Journal of Business & Economic Statistics* 8.1, pp. 27–29.
- Durdu, Bora, Oliver de Groot, and Enrique G Mendoza (2019). “Global v. Local Methods in the Quantitative Analysis of Open-Economy Models with Incomplete Markets”. In: *Mimeo*.
- Guerrieri, Luca and Matteo Iacoviello (2015). “OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily”. In: *Journal of Monetary Economics* 70, pp. 22–38.
- Holden, Thomas (2016). “Computation of solutions to dynamic models with occasionally binding constraints”. In: *EconStor Preprints* 144569.
- Huggett, Mark (1993). “The risk-free rate in heterogeneous-agent incomplete-insurance economies”. In: *Journal of economic Dynamics and Control* 17.5-6, pp. 953–969.
- Judd, Kenneth L (1998). *Numerical methods in economics*. MIT press.
- Ljungqvist, Lars and Thomas J Sargent (2012). *Recursive macroeconomic theory*. MIT press.
- Ludwig, Alexander and Matthias Schön (2018). “Endogenous grids in higher dimensions: Delaunay interpolation and hybrid methods”. In: *Computational Economics* 51.3, pp. 463–492.
- Mendoza, Enrique G (1995). “The terms of trade, the real exchange rate, and economic fluctuations”. In: *International Economic Review*, pp. 101–137.
- (2010). “Sudden stops, financial crises, and leverage”. In: *The American Economic Review* 100.5, pp. 1941–1966.
- Mendoza, Enrique G and Katherine A Smith (2006). “Quantitative implications of a debt-deflation theory of sudden stops and asset prices”. In: *Journal of International Economics* 70.1, pp. 82–114.

Perri, Fabrizio and Vincenzo Quadrini (2018). “International recessions”. In: *American Economic Review* 108.4-5, pp. 935–84.

Rendahl, Pontus (2015). “Inequality Constraints and Euler Equation-based Solution Methods”. In: *The Economic Journal* 125.585, pp. 1110–1135.

3.A Appendix

3.A.1 Introduction

This Appendix provides a user’s guide for the Matlab codes that implement the *FiPIt* algorithm. It describes how the various steps of the algorithm presented in Section 3.3 of the paper are undertaken in the computer programs. The programs are available at sergiovallalvazo.com in a zip file labeled *MendozaVillalvazoFiPItCode* available online. The main directory of this file has the same name, and it contains two folders named *FiPIt* and *Mfiles*. The main Matlab script is named *mainFiPIt.m* and is located in the *FiPIt* folder. This folder also includes the output files as well as script files used to generate various output components (moments, graphs, etc.). The *mainFiPIt.m* program calls several function scripts that are stored in the *MFiles* folder. Table 3.8 provides a list of all the files, their location and contents.

The output of *mainFiPIt.m* is stored in a .mat file. To solve the variant of the model in which the credit constraint never binds (denoted the RBC model), set the value of κ high enough so as to ensure that this is the case. Under our calibration, $\kappa > 1$ is sufficient. The .mat file with the RBC solution is named *solFiPIt.RBC.mat*. To solve the Sudden Stops (SS) model, set $\kappa < 1$. The .mat file with this solution is named *solFiPIt_SS.mat*. The long-run moments of these two models reported in Table 3.2 and 3.6.e of the paper are computed using *script1_Moments.m*, choosing to comment in or out either line 14 or 15 to load the corresponding .mat file with the RBC or

Table 3.8: Files Included in the *MendozaVillalvazoFiPhtCode* Directory

<i>Name</i>	<i>Folder Location</i>	<i>Description</i>
mainFiPht.m	FiPht/	Main script
script1_Moments.m	FiPht/	Script to obtain moments in Table 3.2 and 3.6.e
script2_PolicyPlot.m	FiPht/	Script to produce the Figures
script4_TableDiffAmpl.m	FiPht/	Script to the probability of Sudden Stops
fBiLinearInterpolation.m	MFiles/	Function for bi-linear interpolation
fFiPht_Cons.m	MFiles/	Function to find consumption rule from the Euler Equation for bonds using FiPht
fFiPht_EulerError.m	MFiles/	Function to compute Euler Errors
fFiPht_MuBond.m	MFiles/	Function to compute Lagrange multiplier of ad-hoc debt limit in the RBC model
fFiPht_MuHat.m	MFiles/	Function that solves for credit constraint multiplier ratio using a non-linear solver
fFiPht_PriceK.m	MFiles/	Function to compute price of capital from the capital Euler Equation using FiPht
fMarkov.m	MFiles/	Function to generate Markov chain simulation

SS solution. Similarly, to produce the policy function plots run `script2_PolicyPlot.m` and to produce the probability of Sudden Stops run `script4_TableDiffAmpl.m`.

3.A.2 Recursive Equilibrium Conditions

To implement the *FiPIt* method, we first re-write the equilibrium conditions of the model in recursive form. The model has two endogenous states, b and k , and three exogenous states, using s to denote the triple of exogenous shocks $s \equiv (A, R, p)$, which includes the shocks to TFP (A), the world interest rate (R) and the price of imported inputs (p). The recursive equilibrium is defined by a set of recursive functions for allocations $[b'(b, k, s), k'(b, k, s), c(b, k, s), L(b, k, s), v(b, k, s)]$, prices $[w(b, k, s), q(b, k, s), d(b, k, s)]$ and multipliers $[\lambda(b, k, s), \mu(b, k, s)]$ that satisfy the following recursive equilibrium conditions:

$$\left(c(b, k, s) - \frac{L(b, k, s)^\omega}{\omega} \right)^{-\sigma} = \lambda(b, k, s)(1 + \tau) \quad (3.48)$$

$$\alpha A k^\gamma L(b, k, s)^{\alpha-1} v(b, k, s)^\eta = w(b, k, s) \left(1 + \phi(R - 1) + \frac{\mu(b, k, s)}{\lambda(b, k, s)} \phi R \right) \quad (3.49)$$

$$\eta A k^\gamma L(b, k, s)^\alpha v(b, k, s)^{\eta-1} = p \left(1 + \phi(R - 1) + \frac{\mu(b, k, s)}{\lambda(b, k, s)} \phi R \right) \quad (3.50)$$

$$\lambda(b, k, s) = R\beta E[\lambda(b'(b, k, s), k'(b, k, s), s')] + \mu(b, k, s) \quad (3.51)$$

$$\begin{aligned}
\lambda(b, k, s) = & \\
& \frac{\beta}{q(b, k, s)} E [\lambda(b'(b, k, s), k'(b, k, s), s')(d(b'(b, k, s), k'(b, k, s), s') + q'(b'(b, k, s), k'(b, k, s), s')))] \\
& + \mu(b, k, s)\kappa
\end{aligned} \tag{3.52}$$

$$d(b, k, s) = \gamma A k^{\gamma-1} L(b, k, s)^\alpha v(b, k, s)^\eta - \delta + \frac{a (k'(b, k, s) - k)^2}{2 k^2} \tag{3.53}$$

$$q(b, k, s) = 1 + a \left(\frac{k'(b, k, s) - k}{k} \right) \tag{3.54}$$

$$w(b, k, s) = L(b, k, s)^{\omega-1} (1 + \tau) \tag{3.55}$$

$$\begin{aligned}
c(b, k, s)(1 + \tau) + k'(b, k, s) - (1 - \delta)k + \frac{a (k'(b, k, s) - k)^2}{2 k} &= A k^\gamma L(b, k, s)^\alpha v(b, k, s)^\eta \\
- p v(b, k, s) - \phi(R - 1)(L(b, k, s)^\omega (1 + \tau) + p v(b, k, s)) - R^{-1} b'(b, k, s) + b &
\end{aligned} \tag{3.56}$$

3.A.3 Contents of the mainFiPIt.m program

The mainFiPIt.m file is divided into 5 cells, each one including comments describing how the contents of each cell relate to each of the seven algorithm steps described in Section 3.3.2 of the dissertation. The itemized step numbers labeled in bold typeface below match the step numbers in the paper description, with the line in the Matlab code in which the step is executed indicated in parenthesis.

Cell 1. Parameterization & State Space: Sets the model's parameter values, creates the discrete grids of bonds and capital, defines the Markov processes of shocks, and sets the values of program parameters that define the method to solve for capital price, the convergence criteria, the maximum number of

iterations and the updating coefficients for decision rule conjectures between one iteration and the next. The endogenous states are foreign bonds b and domestic capital k . The exogenous states are included in s , which denotes a triple of shocks $s \equiv (A, R, p)$ that includes TFP (A), the world interest rate (R) and the price of imported inputs (p). The realization set for shock triples $s \in \mathbf{S}$ comes from the discretization of the stochastic processes of the shocks, which is typically done using Tauchen's quadrature method. Here, we take \mathbf{S} and the associated Markov transition probability matrix from Mendoza (2010), where each shock has two realizations and hence \mathbf{S} has eight triples. For the endogenous states, the algorithm defines grids with a total of $nBondGrid$ nodes for bonds and $nCapitalGrid$ nodes for capital. The state space has $nBondGrid \times nCapitalGrid \times 8$ elements and is defined by all $(b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$. The conditional statement starting in Line 82 adjusts the bonds grid when the SS model is being solved to make sure the collateral constraint binds before the lower bound of the grid. The recursive equilibrium is defined by a set of recursive functions for allocations $[b'(b, k, s), k'(b, k, s), c(b, k, s), L(b, k, s), v(b, k, s)]$, prices $[w(b, k, s), q(b, k, s), d(b, k, s)]$ and the multipliers $[\lambda(b, k, s), \mu(b, k, s)]$. The model and program parameters are listed in Table 3.9.

Cell 2. Initial Conjectures, Array Definitions & Non-linear Solver Options:

This cell defines the initial conjectures for the equilibrium recursive functions. Following the notation in the paper, at any iteration j the initial conjectured functions are denoted $\hat{q}_j(b, k, s)$ for the price of capital, $\hat{b}'_j(b, k, s)$ for the decision rule for bonds, and $\hat{\mu}_j(b, k, s) \equiv \mu_j(b, k, s)/\lambda_j(b, k, s)$ for the multiplier ratio. This cell also initializes the arrays for other variables and constructs a function that sets the optimization options for the non-linear solver used later in the program to solve for allocations when the credit constraint binds.

Table 3.9: Parameter Values

<i>Calibrated parameters</i>		
σ	coefficient of relative risk aversion	2.0
ω	labor elasticity coefficient	1.8461
β	discount factor	0.92
a	capital adjustment costs coefficient	2.75
ϕ	fraction of input costs requiring working capital	0.2579
δ	depreciation rate	0.088
α	labor share in gross output	0.59
η	imported inputs share in gross output	0.10
γ	capital share in gross output	0.31
τ	tax on consumption	0.17
A	average TFP	6.982
<i>Algorithm parameters</i>		
ρ^b	Updating weight for bonds decision rule	1.00
ρ^μ	Updating weight for multiplier ratio	1.00
ρ^q	Updating weight for price of capital	0.30
ε^f	Function convergence criterion	10e-4

Step 1. (Line 106) Sets the first-iteration recursive function conjectures to $\hat{b}'_0(b, k, s) = b$, $\hat{q}_0(b, k, s) = 1$ and $\hat{\mu}_0(b, k, s) = 0$ for all $(b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$. The instructions after those initialize the arrays for other variables, the first-iteration value of the convergence metric for the recursive functions ($nMaxDif$) and the iterations counter ($nIter$), and they also define the function $pSolverOpt$ to set the options for Matlab's *fsolve* non-linear solver used later in the code when solving for allocations in states in which $\tilde{\mu} > 0$.

Cell 3. Main Loop Executing Iterations on Equilibrium Recursive Functions: The *While* loop starting in line 143 executes the successive iterations on the equilibrium recursive functions for bonds, price of capital and multiplier ratio. The current iteration number (j) is stored in the integer $nIter$, and the value of the convergence metric attained in iteration $nIter$ is stored in $nMaxDif$.

Step 2. (Line 146) Generates decision rules for capital, investment, factor allocations, gross output and consumption in iteration j implied by the conjectures $\hat{q}_j(b, k, s), \hat{b}'_j(b, k, s), \hat{\mu}_j(b, k, s)$:

$$\begin{aligned}
k'_j(b, k, s) &= \frac{k}{a} [\hat{q}_j(b, k, s) - 1 + a] \\
\tilde{i}_j(b, k, s) &= (k'_j(b, k, s) - k) \left[1 + \frac{a}{2} \left(\frac{k'_j(b, k, s) - k}{k} \right) \right] - \delta k \\
v_j(b, k, s) &= \left\{ \frac{Ak^\gamma \eta^{\frac{\omega-\alpha}{\omega}} \frac{\alpha}{1+\tau} \frac{\alpha}{\omega}}{p^{\frac{\omega-\alpha}{\omega}} [1 + \phi(R-1) + \hat{\mu}_j(b, k, s)\phi R]} \right\}^{\frac{\omega}{\omega(1-\eta)-\alpha}} \\
L_j(b, k, s) &= \left\{ \frac{\alpha}{\eta(1+\tau)} p v_j(b, k, s) \right\}^{\frac{1}{\omega}} \\
y_j(b, k, s) &= Ak^\gamma L_j(b, k, s)^\alpha v_j(b, k, s)^\eta \\
(1+\tau)c_j(b, k, s) &= y_j(b, k, s) - p v_j(b, k, s) - \phi(R-1)[(1+\tau)L_j(b, k, s)^\omega \\
&\quad + p v_j(b, k, s)] - \tilde{i}_j(b, k, s) - \frac{\hat{b}'_j(b, k, s)}{R} + b
\end{aligned}$$

The code uses here the same set of expressions for the RBC and SS solutions. For the latter, the values of factor allocations, gross output and consumption vary with $\tilde{\mu}(\cdot)$, whereas in the RBC solution they do not because $\tilde{\mu}(\cdot) = 0$ always. Note also that since $\tilde{\mu}(\cdot)$ is always set to zero in the first iteration, the first-iteration results of this step are identical when solving either the RBC or SS models. When solving the RBC model, $\tilde{\mu}(\cdot)$ remains zero in all iterations, but when solving the SS model, $\tilde{\mu}(\cdot) > 0$ in states in which the credit constraint binds.

Step 3.1 (Line 166) Assume the collateral constraint does not bind. Solve for new decision rules (indexed $j+1$) for labor, intermediate goods and output. Since the constraint is assumed to be non-binding, these decision rules are the

same in RBC and SS solutions:

$$\begin{aligned}
v_{j+1}(b, k, s) &= \left\{ \frac{Ak^\gamma \eta^{\frac{\omega-\alpha}{\omega}} \frac{\alpha}{1+\tau} \frac{\alpha}{\omega}}{p^{\frac{\omega-\alpha}{\omega}} [1 + \phi(R-1)]} \right\}^{\frac{\omega}{\omega(1-\eta)-\alpha}} \\
L_{j+1}(b, k, s) &= \left\{ \frac{\alpha}{\eta(1+\tau)} p v_{j+1}(b, k, s) \right\}^{\frac{1}{\omega}} \\
y_{j+1}(b, k, s) &= Ak^\gamma L_{j+1}(b, k, s)^\alpha v_{j+1}(b, k, s)^\eta
\end{aligned}$$

Steps 3.2 & 3.3. (Line 181) Solve for the $j+1$ consumption and bonds decision rules using the bonds' Euler Equation and the resource constraint. For each (b, k, s) in the state space, consumption is solved for using the `fFiPIt_Cons.m` function located in the *Mfiles* folder. This function finds the new consumption rule by solving “directly” from the Euler equation, as explained in Step 3.2 of the algorithm description in the paper:

$$\begin{aligned}
c_{j+1}(b, k, s) &= \\
&\left\{ \beta RE \left[\left(c_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s') - \frac{L_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')^\omega}{\omega} \right)^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}} + \\
&\frac{L_{j+1}(b, k, s)^\omega}{\omega},
\end{aligned}$$

`fFiPIt_Cons.m` calls the function `fBiLinearInterpolation.m`, also in the *Mfiles* folder, in order to find the values of $c_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')$ and $L_j(\hat{b}'_j(b, k, s), k'_j(b, k, s), s')$, which are determined using bi-linear interpolation because $\hat{b}'_j(b, k, s)$ and $k'_j(b, k, s)$ are not on the nodes of the bonds and capital grids in general. Once $c_{j+1}(b, k, s)$ is determined, the new bonds policy function $b'_{j+1}(b, k, s)$ is solved for using the resource constraint, and the implied leverage ratio is computed (i.e. the value of $\frac{-q_t^b b_{t+1} + \phi R_t (w_t L_t + p_t v_t)}{\kappa q_t k_{t+1}}$).

Step 3.4. (Line 192) Check if collateral constraint binds using the new decision

(j+1-indexed) decision rules:

$$\frac{b'_{j+1}(b, k, s)}{R} - \phi R [(1 + \tau)L_{j+1}(b, k, s)^\omega + pv_{j+1}(b, k, s)] + \kappa \hat{q}_j(b, k, s)k'_j(b, k, s) \geq 0$$

Line 202 evaluates if there are (b, k, s) states for which the new bonds decision rule is below the lower bound of the bonds grid. In these cases, the lower bound is a binding ad-hoc debt limit. The bonds decision rule is re-set equal to this debt limit, the consumption decision rule is re-set to the value implied by the resource constraint, and we also compute the associated Lagrange multiplier for the binding ad-hoc debt limit.

Step 4. (Line 217) This step is only executed when solving the SS model and only for states (b, k, s) in which the constraint was found to be binding in Step 3.4, because these are the only states in which the decision rules depend on $\tilde{\mu}$. This step solves for $\tilde{\mu}_{j+1}(b, k, s)$ by applying Matlab's *fsolve* root finder to a function formed using the `fFiPIt_MuHat.m` script located in the *Mfiles* folder. `fFiPIt_MuHat.m` forms Equation 3.38 in the paper. It uses the j -indexed functions for consumption and labor to form the expected value in the right-hand-side of eq. (38), which requires the same bi-linear interpolation method used to solve for c_{j+1} in step 3.2. The solver uses the optimization options set in *pSolverOpt* as defined in Cell 1 and returns the value of $\tilde{\mu}_{j+1}(b, k, s)$. The solver uses these options: `optimoptions('fsolve','Display','off','TolFun',1e-18)`. A small tolerance convergence criterion is needed in order to attain convergence of the recursive functions and small Euler errors. We use as initial condition (*vInitX*) the current iteration's initial conjecture $\tilde{\mu}_j(b, k, s)$. After $\tilde{\mu}_{j+1}(b, k, s)$ is solved for, we compute the associated j+1 values of the decision rules using eqs. (33)-(36) in the paper. Keep in mind that there are many variations of occasionally binding

constraints for which the constrained allocations and the multiplier of the binding constraint can be solved separately, in which case there is no need to use a non-linear solver in this step. Two cases explored in the paper are one in which working capital is removed from the collateral constraint and one in which the credit constraint is set to a constant value instead of the value of collateral (see p. 15 and p. 24 of the paper). This makes the *FiPIt* algorithm significantly faster.

Step 5. (Line 250) This step is just a comment noting that at this point in the code we have solved the new ($j+1$ -indexed) optimal decision rules for all (b, k, s) in the state space conditional on the conjectured $\hat{q}_j(b, k, s)$ function.

Step 6. (Line 252) Compute the new pricing function. This step is coded so as to allow the user to choose one of the two alternatives to compute the pricing function described in Steps 6.1 and 6.2 of the paper. The former uses fixed-point iteration, the latter finds q as the forward solution of the capital Euler equation. The fixed-point iteration (forward) solution is chosen by setting `pFixPointPriceK == 1` (`pFixPointPriceK == 0`) in the algorithm parameters of Cell 1. In both cases, we solve for $q_{j+1}(b, k, s)$ using the `fFiPIt_PriceK.m` script located in the *Mfiles* folder. This script solves the following equation, which is Equation 3.41 in the paper (we use (\cdot) to denote (b, k, s) so as to shorten the notation):

$$q_{j+1}(b, k, s) = \frac{\beta E_t \left[\left(c_{j+1} \left(b'_{j+1}(\cdot), k'_j(\cdot), s' \right) - \frac{L_{j+1}(b'_{j+1}(\cdot), k'_j(\cdot), s')^\omega}{\omega} \right)^{-\sigma} \left[d' \left(b'_{j+1}(\cdot), k'_j(\cdot), s' \right) + \hat{q}_j \left(b'_{j+1}(\cdot), k'_j(\cdot), s' \right) \right] \right]}{\left(c_{j+1}(\cdot) - \frac{L_{j+1}(\cdot)^\omega}{\omega} \right)^{-\sigma} (1 - \kappa \tilde{\mu}_{j+1}(\cdot))}$$

where

$$d' (b'_{j+1}(\cdot), k'_j(\cdot), s') = \gamma A' k'_j(\cdot)^{\gamma-1} L_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')^\alpha v_{j+1} (b'_{j+1}(\cdot), k'_j(\cdot), s')^\eta - \delta + \frac{a}{2} \frac{(k'_j(b'_{j+1}(\cdot), k'_j(\cdot), s') - k'_j(\cdot))^2}{k'_j(\cdot)^2}$$

When solving by fixed-point iteration, the above Euler equation solves directly for $q_{j+1}(\cdot)$, since all the terms in the right-hand-side of the expression are known at this point in the code. The equation is solved once and the solution passed on as the new pricing function. Note that in forming the conditional expectation, we use j -indexed conjectures of the price of capital and the capital decision rule (since their $j + 1$ values are not known), but the rest of the relevant recursive functions are indexed $j + 1$ (since they have been solved for in the previous steps of the algorithm). As before, bi-linear interpolation is used to determine the values of all the functions that have $(b'_{j+1}(\cdot), k'_j(\cdot))$ as arguments (the $t + 1$ variables in the conditional expectation of the Euler equation), since those functions are only known at grid nodes.¹³ When solving by forward solution, `fFiPIt_PriceK.m` is used repeatedly to iterate on the above capital Euler equation until $q_{j+1}(\cdot)$ and $\hat{q}_j(\cdot)$ converge, but keeping all the other functions unchanged. For these iterations, the iteration counter is the integer $nIterInner$, and the value of the convergence metric at iteration $nIterInner$ is denoted $nMaxDifInner$. The convergence criterion is the value assigned to the parameter $nTolInner$ in Cell 1.

Step 6.1. (Line 271) If $pFixPointPriceK = 1$, then the first solution for $q_{j+1}(b, k, s)$ generated for each (b, k, s) using `fFiPIt_PriceK.m` is retained as the new pricing function.

Step 6.2. (Line 274) If $pFixPointPriceK = 0$ (which is executed by the *else*

¹³For evaluating dividends, we found that the algorithm performs better if we interpolate the functions that enter in the definition of dividends individually and then generate the value of dividends, instead of first defining dividends and then interpolating the dividends function.

instruction when $\text{pFixPointPriceK} = 1$ is not valid), then `ffIPIt_PriceK.m` is used to generate the new values of $q_{j+1}(b, k, s)$ for each (b, k, s) as we iterate to convergence on the capital pricing function.

Step 7. (Line 281) Check convergence and update conjectures. The convergence criterion is given by $nMaxDif \leq \varepsilon^f$, where $nMaxDif$ is the following convergence metric:

$$nMaxDif = \max \left\{ |q_{j+1}(b, k, s) - \hat{q}_j(b, k, s)|, |b'_{j+1}(b, k, s) - \hat{b}'_j(b, k, s)|, |\tilde{\mu}_{j+1}(b, k, s) - \hat{\mu}_j(b, k, s)| \right\},$$

$\forall (b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$. The value of ε^f is defined by setting the program parameter $nTol$ in Cell 1. If convergence is attained, the recursive equilibrium has been solved and the results are stored in either the `solFiPIt_SS.mat` file for the SS model or the `solFiPIt_RBC.mat` file for the RBC model. If convergence is not attained, then generate new conjectures as follows:

$$\hat{x}_{j+1}(b, k, s) = (1 - \rho^x) \hat{x}_j(b, k, s) + \rho^x x_{j+1}(b, k, s)$$

for $x = [q, b, \tilde{\mu}]$ and some $0 \leq \rho^x$. $\hat{x}_j(b, k, s)$ in the right-hand-side of this expression represents the initial conjectures used in the current iteration, while $\hat{x}_{j+1}(b, k, s)$ in the left-hand-side denotes the new conjectures for the next iteration. Use $0 < \rho^x < 1$ ($\rho^x > 1$) for the particular function $x(\cdot)$ that is not converging (converging too slowly). The values of the ρ^x coefficients are set with the parameters $nUpdateGuessB$ and $nUpdateGuessPK$ in Cell 1.¹⁴ Return to Step 2 (Line 146) using the new conjectures for the next iteration.

¹⁴We set $\rho^B = \rho^{\tilde{\mu}} = 1$ and $\rho^q = 0.3$ because this produced the best convergence performance, but this can change with other parameterizations or in other applications of the algorithm.

Cell 4. Compute Euler Equation Errors.

The solution of the recursive equilibrium is completed when the program exits Cell 3. The next two cells generate two important objects based on the model solution. First, Cell 4 computes the errors of the Euler equations of bonds and capital using the `fFiPIt.EulerError.m` function located in the *Mfiles* folder. Then Cell 5 computes the ergodic distribution of bonds, capital and shocks. To compute the Euler errors, the Euler equations are evaluated at the equilibrium solutions rather than used to solve for the equilibrium. Hence, `fFiPIt.EulerError.m` uses the equilibrium functions (the last solutions generated by the functions that converged according to the tolerance criterion) in all the relevant terms of the Euler equations.

Cell 5. Compute the Ergodic Distribution.

We compute the ergodic distribution of (b, k, s) by iterating to convergence on the law of motion of the conditional transition probabilities from (b, k, s) (denoted $M_j(b, k, s)$) to (b', k', s') (denoted $M_{j+1}(b', k', s')$) $\forall (b, k, s), (b', k', s') \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$. The initial guess (called *mErgDistGuess* in line 404) is a uniform distribution. The law of motion is formed using the decision rules for capital and bonds and the exogenous Markov process of the shocks. Since we have solved for approximately continuous decision rules using bi-linear interpolation, we use a standard modification of this law of motion adjusted for the fact that decision rules do not yield values on the nodes of the bonds and capital grids in general. For every (b, k, s) we find $b_L \leq b'(b, k, s) \leq b_U$ and $k_L \leq k'(b, k, s) \leq k_U$, where b_L, b_U, k_L, k_U are the grid points closest to $b'(\cdot)$ and $k'(\cdot)$. Then we iterate on the conditional distributions as follows:

$$M_{j+1}(b_L, k_L, s') = \sum_s Pr[s'|s] M_j(b, k, s) \left(\frac{b_U - b'(b, k, s)}{b_U - b_L} \right) \left(\frac{k_U - k'(b, k, s)}{k_U - k_L} \right)$$

$$M_{j+1}(b_L, k_U, s') = \sum_s Pr[s'|s] M_j(b, k, s) \left(\frac{b_U - b'(b, k, s)}{b_U - b_L} \right) \left(\frac{k'(b, k, s) - k_L}{k_U - k_L} \right)$$

$$M_{j+1}(b_U, k_L, s') = \sum_s Pr[s'|s] M_j(b, k, s) \left(\frac{b'(b, k, s) - b_L}{b_U - b_L} \right) \left(\frac{k_U - k'(b, k, s)}{k_U - k_L} \right)$$

$$M_{j+1}(b_U, k_U, s') = \sum_s Pr[s'|s] M_j(b, k, s) \left(\frac{b'(b, k, s) - b_L}{b_U - b_L} \right) \left(\frac{k'(b, k, s) - k_L}{k_U - k_L} \right)$$

The convergence criterion is $\max |M_{j+1}(b, k, s) - M_j(b, k, s)| < \epsilon_{Dist} \quad \forall (b, k, s) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{S}$, with the value of ϵ_{Dist} set by the parameter *nTolDist* in Cell 1.

3.A.4 Auxiliary Notes

- **Interpolation:** Bi-linear interpolation can be done using the “interp2” Matlab function, but we found that programming the interpolation directly improved the performance of the code. We determine first the interpolation nodes, and then apply the standard bi-linear interpolation rule. The scripts that implement the functions interpolations determine the relevant interpolation nodes and then perform the bi-linear interpolation. To determine the interpolation nodes, for each (b, k, s) , create first vectors with the differences $h^b(b, k, s) = \hat{b}'_j(b, k, s) - b$ and $h^k(b, k, s) = k'_j(b, k, s) - k$, then find the location of the smallest positive difference and smallest negative difference (i.e. the difference closest to zero from below) in these vectors. For example, for the interpolation nodes over the b dimension (b_n, b_{n+1}) , find the locations of $\text{argmin} h^b(b, k, s)$ for $h^b(b, k, s) \geq 0$ and $\text{argmax} h^b(b, k, s)$ for $h^b(b, k, s) \leq 0$. b_n is the location of the argmin and b_{n+1} is the location of the argmax. Once the interpolation nodes are found, the interpolation is executed by calling the `fBiLinearInterpolation.m` function

located in the *Mfiles* folder. The scripts also make these adjustments when the interpolated functions return decision rule values outside the state space: Use extrapolation if $k'_j(b, k, s)$ returns a value below (above) the first node k^1 (last node $k^{N_{CapitalGrid}}$) and also if $\hat{b}'_j(b, k, s)$ returns a value above the last node $b^{N_{BondGrid}}$, but for $\hat{b}'_j(b, k, s) < b^1$ evaluate the functions at b^1 , because the lower bound on bonds represents an ad-hoc debt limit used for calibration.

- Parallelization:** There are several loops that run faster in parallel, using *parfor* instead of *for*. This can be done with all loops that do not need to run sequentially. The outmost loop controlling the iterations of the policy and pricing functions needs to be executed sequentially, but several others can be parallelized. *Parfor* can be used in Step 2, 3, 4, and 6. For the *FiPIt* variant of Step 6 a sequential sum is needed to attain convergence. We included comments in the code indicating specific loops where *parfor* was used. Using *parfor* requires Matlab's Parallel Computing Toolbox. Note also that setting the number of workers to the largest feasible (i.e. the number of processors) does not necessarily minimize execution time, particularly in machines with several processors. In various computers with more than 16 processors, we found that using 6 or 7 workers produced the fastest execution times.
- Invalid allocations:** Rule out allocations with non-positive arguments in the utility function. These are cases such that, at any iteration and for a given triple (b, k, s) the conjectured functions (indexed by j) or the unconstrained or constrained new decision rules (indexed by $j + 1$) yield $c - L^\omega/\omega \leq 0$. In these cases, the solution of consumption when the constraint does not bind and/or of the multiplier $\tilde{\mu}$ when it binds cannot be obtained because they involve the fractional exponent $1/\sigma$ (for $\sigma > 1$), which requires a positive base. Note

that this requirement is stricter than feasibility, because it is not just that the allocations are technologically feasible, they also need to avoid hitting the Inada condition of the CRRA utility function. In the *mainFiPit.m* program, this causes an error that stops execution at the point in which the first attempt to solve for a state of nature with $c - L^\omega/\omega \leq 0$ is encountered. As explained in the paper (see p. 13 and p. 15), however, the *FiPit* algorithm has the advantage that starting from initial conjectures $\hat{b}'_0(b, k, s)$, $\hat{q}_0(b, k, s)$, $\hat{\mu}_0(b, k, s)$ such that the implied labor and consumption decision rules satisfy $c_0(\cdot) - \frac{L_0(\cdot)^\omega}{\omega} > 0$, implies that $c_j(\cdot) - \frac{L_j(\cdot)^\omega}{\omega} > 0$ for any iteration $j > 0$. For the baseline calibration and all six variations we solved for, the initial conditions $\hat{b}'_0(b, k, s) = b$, $\hat{q}_0(b, k, s) = 1$ and $\hat{\mu}_0(b, k, s) = 0$ satisfied this condition.

3.A.5 Sketch of Other *FiPit* Applications

We provide here a brief sketch of four examples:

1. Mendoza (1995): This is an RBC small open economy with incomplete markets and three sectors, quadratic capital adjustment costs given by $(\phi/2)(k_{t+1} - k_t)^2$, and a maximum debt limit as the only occasionally binding constraint. The model has endogenous discounting, but consider a variant with a standard constant discount factor. The adjustment costs formulation does not satisfy the Hayashi conditions required for the average and marginal Tobin's Q to be the same, but for implementing *FiPit* define a quasi capital pricing function given by $q_t \equiv 1 + \phi(k_{t+1} - k_t)$, so that given a conjecture of this pricing function we can obtain an implied capital decision rule. Start with this pricing conjecture and a conjectured bonds decision rule. The model's equilibrium conditions, the implied capital decision rule,

and the bonds decision rule can be used so that the resource constraint for tradable goods yields an implied decision rule for tradables consumption. *FiPIt* can then be used on the Euler equation for bonds to solve for a new tradables consumption decision rule, and the resource constraint yields a new bonds decision rule. *FiPIt* can then be applied to the Euler equation for capital to solve for a new q function.

2. Ludwig and Schön (2018): This is a model of optimal human capital accumulation h with a no-borrowing constraint on an asset a that pays an exogenous interest rate R (i.e. a small open economy). Human capital depreciates at rate δ and is produced with a concave function of human capital investment $f(i)$. Agents have CRRA period utility and an exogenous probability of survival given by an increasing, concave function $s(h)$. To solve using *FiPIt*, start with a conjectured decision rule for assets $\hat{A}(a, h)$ and a conjecture for the shadow relative price of human capital investment $\hat{\mu}(a, h)$ where $\mu \equiv \mu/\lambda$ and λ and μ are the multipliers on the resource constraint and law of motion of human capital accumulation respectively. Given these conjectures, the model's equilibrium conditions yield implied decision rules for human capital, consumption and investment in human capital. Then *FiPIt* can be applied to the Euler equation on assets to solve for a new consumption decision rule and using the result in the resource constraint yields a new decision rule for assets $A(a, h)$. If $A(a, h) < 0$, redefine the decision rule as $A(a, h) = 0$, set the associated consumption to the amount supported by the resource constraint, and compute the ratio ψ/λ (where ψ is the multiplier on the no-borrowing constraint). Finally, rewrite the Euler equation for human capital in terms of the ratio μ/λ and apply *FiPIt* to solve for a new decision rule for $\tilde{\mu}$. There is no need to use

a root-finder in this case.

3. Mendoza and Smith (2006): This is a stochastic model of a small open economy in which agents trade in world bond markets and in a market where equity on the economy's capital can be bought by foreign investors, who face a quadratic cost of purchasing equity. A productivity shock affects equity returns. There is a credit constraint imposing a limit on the ratio of debt to the market value of the equity holdings of domestic agents, and a short-selling limit on the equity position. Given conjectures of the decision rule for bonds and the equity pricing function, the optimality condition of foreign investors and market clearing conditions yield an implied decision rule for equity holdings (the quadratic adjustment cost plays a role similar to the capital adjustment cost in the SS model we solved earlier). Given these, the resource constraint of the small open economy yields a decision rule for consumption. Assuming the credit constraints do not bind, *FiPIt* can then be applied to the bonds Euler equation to solve for a new consumption decision rule, and the resource constraint yields a new bonds decision rule. If the latter yields a value that violates the credit constraint, the constraint is imposed with equality to obtain new values for the bonds decision rule and consumption, and for the ratio of the multiplier of the borrowing constraint. Finally, *FiPIt* is applied to the Euler equation for equity holdings to obtain a new equity pricing function.
4. Huggett (1993): This is one of the canonical heterogeneous agents models in which a continuum of agents trade non-state-contingent debt facing idiosyncratic Markov income shocks and a maximum debt limit. The optimization problem solved by an individual agent, who takes an exogenously-determined value of the interest rate as given, is identical to that of the

small open endowment economy studied in Section 3.2, which has only one endogenous state variable. Start with a conjectured decision rule for bonds, use the resource constraint to obtain the implied decision rule for consumption. Then apply *FiPIt* to solve for a new consumption decision rule, and use the resource constraint to obtain a new bonds decision rule. If the latter violates the maximum debt limit, redefine to bonds decision rule to match the debt limit and set the associated consumption decision rule to the amount supported by the resource constraint. Iterate to convergence on the bonds decision rule and then use the decision rules and Markov process of income shocks to compute the ergodic distribution of bonds and income (i.e. the wealth and income distribution across agents). The difference with the small open economy is that now the interest rate is also part of the solution. The ergodic distribution is used to compute the aggregate demand for bonds (i.e. the mean of asset demand across agents), which must be equal zero at equilibrium in order to clear the bond market. If it yields exceeds demand (supply), the interest rate is reduced (increased) until the market-clearing condition holds up to a convergence criterion. Again *FiPIt* does not require a root finder.